NORTH ATLANTIC TREATY ORGANISATION

RESEARCH AND TECHNOLOGY ORGANISATION

AC/323(AVT-039)TP/103

www.rta.nato.int

RTO AGARDograph

AG-AVT-039

Unsteady Motions in Combustion Chambers for Propulsion Systems

(Mouvements instables dans les chambres de combustion des systèmes de propulsion)

This AGARDograph was approved by the Propulsion and Energetics Panel (PEP) of NATO's Advisory Group for Aerospace Research and Development (AGARD); subsequently it has been sponsored by the Applied Vehicle Technology Panel (AVT) of NATO's Research and Technology Organisation (RTO).

Published December 2006

Distribution and Availability on Back Cover

NORTH ATLANTIC TREATY ORGANISATION

RESEARCH AND TECHNOLOGY ORGANISATION

AC/323(AVT-039)TP/103

www.rta.nato.int

RTO AGARDograph **AG-AVT-039**

Unsteady Motions in Combustion Chambers for Propulsion Systems

(Mouvements instables dans les chambres de combustion des systèmes de propulsion)

This AGARDograph was approved by the Propulsion and Energetics Panel (PEP) of NATO's Advisory Group for Aerospace Research and Development (AGARD); subsequently it has been sponsored by the Applied Vehicle Technology Panel (AVT) of NATO's Research and Technology Organisation (RTO).

F.E.C. Culick

California Institute of Technology

The Research and Technology Organisation (RTO) of NATO

RTO is the single focus in NATO for Defence Research and Technology activities. Its mission is to conduct and promote co-operative research and information exchange. The objective is to support the development and effective use of national defence research and technology and to meet the military needs of the Alliance, to maintain a technological lead, and to provide advice to NATO and national decision makers. The RTO performs its mission with the support of an extensive network of national experts. It also ensures effective co-ordination with other NATO bodies involved in R&T activities.

RTO reports both to the Military Committee of NATO and to the Conference of National Armament Directors. It comprises a Research and Technology Board (RTB) as the highest level of national representation and the Research and Technology Agency (RTA), a dedicated staff with its headquarters in Neuilly, near Paris, France. In order to facilitate contacts with the military users and other NATO activities, a small part of the RTA staff is located in NATO Headquarters in Brussels. The Brussels staff also co-ordinates RTO's co-operation with nations in Middle and Eastern Europe, to which RTO attaches particular importance especially as working together in the field of research is one of the more promising areas of co-operation.

The total spectrum of R&T activities is covered by the following 7 bodies:

- AVT Applied Vehicle Technology Panel
- HFM Human Factors and Medicine Panel
- IST Information Systems Technology Panel
- NMSG NATO Modelling and Simulation Group
- SAS System Analysis and Studies Panel
- SCI Systems Concepts and Integration Panel
- SET Sensors and Electronics Technology Panel

These bodies are made up of national representatives as well as generally recognised 'world class' scientists. They also provide a communication link to military users and other NATO bodies. RTO's scientific and technological work is carried out by Technical Teams, created for specific activities and with a specific duration. Such Technical Teams can organise workshops, symposia, field trials, lecture series and training courses. An important function of these Technical Teams is to ensure the continuity of the expert networks.

RTO builds upon earlier co-operation in defence research and technology as set-up under the Advisory Group for Aerospace Research and Development (AGARD) and the Defence Research Group (DRG). AGARD and the DRG share common roots in that they were both established at the initiative of Dr Theodore von Kármán, a leading aerospace scientist, who early on recognised the importance of scientific support for the Allied Armed Forces. RTO is capitalising on these common roots in order to provide the Alliance and the NATO nations with a strong scientific and technological basis that will guarantee a solid base for the future.

The content of this publication has been reproduced
directly from material supplied by RTO or the authors.

Published December 2006

Copyright © RTO/NATO 2006
All Rights Reserved

ISBNs 92-837-0059-7 / 978-92-837-0059-3

Single copies of this publication or of a part of it may be made for individual use only. The approval of the RTA Information Management Systems Branch is required for more than one copy to be made or an extract included in another publication. Requests to do so should be sent to the address on the back cover.

Table of Contents

	Page
List of Figures	**ix**
List of Tables	**xx**
Foreword	**xxi**
Avant-propos	**xxii**
Executive Summary and Synthèse	**ES-1**
Overview	**O-1**
Chapter 1 – Combustion Instabilities in Propulsion Systems	**1-1**
1.1 Introduction	1-2
1.2 Historical Background	1-5
1.2.1 Liquid and Gas-Fueled Rockets	1-7
1.2.2 Combustion Instabilities in Thrust Augmentors	1-13
1.2.3 Combustion Instabilities in Ramjet Engines	1-15
1.2.4 Combustion Instabilities in Gas Turbines	1-17
1.2.5 Combustion Instabilities in Solid Propellant Rocket Motors	1-17
1.3 Mechanisms of Combustion Instabilities	1-22
1.3.1 Mechanisms in Liquid-Fueled Rockets	1-22
1.3.2 Sensitivity of Combustion Processes; Time Lags	1-24
1.3.3 Vortex Shedding and Vortex/Mean Flow Interactions	1-25
1.3.4 Operation Near the Lean Blow-Out Limit	1-27
1.3.5 Mechanisms in Solid-Fueled Rockets	1-28
1.3.6 Combustion Dynamics	1-28
1.4 Physical Characteristics of Combustion Instabilities	1-29
1.5 Linear Behavior	1-33
1.5.1 Gains and Losses of Acoustic Energy; Linear Stability	1-37
1.6 Nonlinear Behavior	1-41
1.6.1 Linear Behavior Interpreted as the Motion of a Simple Oscillator	1-42
1.6.2 Combustion Dynamics and Stability	1-44
1.6.3 Nonlinear Behavior Interpreted as the Motion of a Nonlinear Oscillator	1-45
1.7 Analysis and Numerical Simulations of Combustion Instabilities	1-49
1.8 A Précis of the Book	1-52
Chapter 2 – Combustion Dynamics and Mechanisms of Combustion Instabilities	**2-1**
2.1 Mechanisms of Instabilities in Solid Propellant Rocket Motors	2-2

2.1.1 Qualitative Interpretation of the Basic Mechanism 2-3
2.1.2 Early Historical Background Leading to the QSHOD Model 2-6
2.2 Analysis of the QSHOD Model 2-18
2.2.1 Estimates of Some Characteristic Lengths and Times 2-19
2.2.2 Calculation of the Response Function 2-22
2.2.3 Measurements of the Response Function; Comparison of Experimental Results and the QSHOD Model 2-30
2.2.4 The Zel'dovich-Novozhilov (Z-N) Model 2-34
2.2.5 Revisions and Extensions of the QSHOD Model 2-35
2.2.6 Some Results for a Special Extension of the QSHOD Model 2-36
2.2.7 Some Results for the Combustion Response Function 2-39
2.2.8 The Combustion Response and Possible Sensitivity of Global Dynamics to Velocity Coupling 2-40
2.2.9 Generation of Vorticity and Vortex Shedding 2-44
2.2.10 Distributed Combustion 2-51
2.3 Mechanisms of Combustion Instabilities in Liquid-Fueled Systems 2-52
2.3.1 Atomization, Droplet Vaporization and Burning 2-52
2.3.2 Interpretation with a Time Lag 2-55
2.3.3 Convective Waves 2-59
2.3.4 Vortex Shedding from Rearward-Facing Steps 2-62
2.3.5 A Model for Vortex Shedding and Excitation of Acoustic Waves 2-70
2.4 Further Remarks on Particular Forms of Liquid-Fueled Systems 2-72
2.4.1 Combustion Instabilities in Liquid Rockets 2-73
2.4.2 Application of the Time Lag Model to Gas and Liquid Rockets 2-75
2.4.3 Pogo Instabilities 2-79
2.4.4 Combustion Instabilities in Thrust Augmentors 2-80
2.4.5 Combustion Instabilities in Ramjet Engines 2-86
2.4.6 Unsteady Behavior of the Inlet/Diffuser 2-87
2.4.7 The Time Lag Model Applied to Combustion Instabilities in Ramjet Engines 2-90
2.5 Dynamics of Flames and Flame Sheets as a Mechanism 2-91
2.6 Fluctuations of Mixture Ratio as a Mechanism 2-92
2.7 The Rijke Tube: Simplest Example of Thermoacoustic Instabilities 2-94
2.7.1 The Electrically Driven Rijke Tube 2-95
2.7.2 Mean Field in the Rijke Tube 2-101
2.7.3 Acoustic Field in the Rijke Tube 2-102
2.7.4 Acoustic Field and Stability by Matching Waves 2-103
2.7.5 Stability Analyzed by a Method of Spatial Averaging 2-106
2.7.6 Nonlinear Behavior due to Rectification 2-110
2.7.7 A Simple Analysis of the Flame-Driven Rijke Tube 2-111

Chapter 3 – Equations for Unsteady Motions in Combustion Chambers 3-1
3.1 Modes of Wave Motion in a Compressible Medium 3-1
3.2 Equations of Motion in a Reacting Flow 3-2
3.3 Two-Parameter Expansion of the Equations of Motion 3-4
3.3.1 Expansion in Mean and Fluctuating Values 3-5
3.3.2 Equations for the Mean Flow 3-7

3.3.3 Systems of Equations for the Fluctuations 3-8
3.4 Nonlinear Wave Equations for the Pressure Field 3-11

Chapter 4 – Modal Expansion and Spatial Averaging; An Iterative Method of Solution 4-1
4.1 Application of a Green's Function for Steady Waves 4-2
4.1.1 Approximate Solution by Iteration 4-6
4.2 An Alternative Derivation of the First Order Formula 4-7
4.3 Approximate Solution for Unsteady Nonlinear Motions 4-8
4.4 An Alternative Application of Spatial Averaging: The Method of Least Residuals 4-11
4.5 Application of Time-Averaging 4-13
4.6 The Procedure for Iterative Solution to the Oscillator Equations 4-17
4.6.1 Iteration on the Mach Number of the Mean Flow 4-18
4.6.2 Zeroth and First Order Solutions for the Eigenvalues and Basis Functions 4-20

Chapter 5 – Some Fundamentals of Acoustics 5-1
5.1 The Linearized Equations of Motion; The Velocity Potential 5-1
5.1.1 The Velocity Potential 5-4
5.2 Elementary Solutions to the Linear Wave Equation 5-5
5.2.1 Plane Waves 5-5
5.2.2 Spherical Waves 5-8
5.2.3 Cylindrical Waves 5-10
5.3 An Estimate of the Influence of Internal Heat Conduction on the Propagation of Acoustic Waves 5-10
5.4 Energy and Intensity Associated with Acoustic Waves 5-13
5.5 The Growth or Decay Constant 5-16
5.6 Boundary Conditions: Reflections from a Surface 5-17
5.6.1 Reflections of Plane Waves at a Surface 5-18
5.7 Wave Propagation in Tubes; Normal Modes 5-20
5.7.1 Waves in Closed Tubes 5-20
5.7.2 Normal Modes for Tubes Having Discontinuities of Cross-Sectional Area 5-22
5.8 Normal Acoustic Modes and Frequencies for a Chamber 5-24
5.8.1 Normal Modes for Rectangular Chambers 5-24
5.8.2 Normal Modes for a Circular Cylindrical Chamber 5-25
5.9 Viscous Losses at an Inert Surface 5-26
5.9.1 Dissipation of Energy Within the Acoustic Boundary Layer 5-27
5.9.2 Another Way of Computing the Decay Constant 5-29
5.9.3 Still Another Way of Computing the Decay Constant 5-31
5.9.4 First Order Correction to the Mode Shape 5-33
5.10 Propagation of Higher Modes in Tubes; Cut-off Frequencies 5-36
5.11 The Impedance Tube 5-37

Chapter 6 – Linear Stability of Combustor Dynamics 6-1
6.1 Historical Background of Linear Stability 6-1

6.2 Zero-Dimensional Instability of a Bulk Mode 6-4
6.3 A Formal Solution to the Problem of Linear Stability 6-13
6.4 An Alternative Calculation of Linear Stability 6-15
6.5 Linear Stability with a Heat Source and Motion of the Boundary 6-16
6.6 Rayleigh's Criterion and Linear Stability 6-18
6.7 Some Results for Linear Stability in Three Dimensions 6-21
6.7.1 Linear Stability of Three-Dimensional Motions 6-22
6.7.2 The Admittance and Response Functions for a Burning Surface 6-27
6.7.3 The First Measurements of the Stability Boundary for a Solid Rocket 6-28
6.8 Stability of Oscillations in a Bulk Mode 6-30
6.9 Linear Stability in the One-Dimensional Approximation 6-32
6.9.1 The One-Dimensional Approximation to 'Flow-Turning' 6-35
6.9.2 The One-Dimensional Approximation to 'Pumping' 6-38
6.9.3 Stability of a Simple *T*-Burner 6-44
6.10 Combined Three-Dimensional and One-Dimensional Results 6-46
6.11 An Example of Linear Stability for a Solid Rocket 6-48
6.12 Vorticity and Stability in Solid Propellant Rockets 6-52
6.12.1 Generation of Large Vortices; Parietal Vortex Shedding 6-53
6.13 Predictions of Stability Boundaries for Liquid Rockets 6-57
6.14 Contributions to the Growth Constant for Linear Stability 6-58
6.14.1 Mean Flow/Acoustics Interactions 6-58
6.14.2 Attenuation by a Choked Exhaust Nozzle 6-59
6.14.3 Attenuation of Acoustic Energy Due to Condensed Products of Combustion 6-66
6.15 Distributed Combustion 6-72

Chapter 7 – Nonlinear Behavior of Combustor Dynamics 7-1
7.1 Early Works on Nonlinear Combustion Instabilities 7-3
7.2 A Single Nonlinear Mode 7-11
7.3 The Two-Mode Approximation 7-18
7.3.1 Existence of Limit Cycles 7-20
7.3.2 Stability of Limit Cycles 7-20
7.4 Transverse Modes and the Method of Averaging 7-23
7.4.1 Periodic Limit Cycles for Transverse Modes 7-25
7.5 Observations of a Spinning Transverse Mode 7-27
7.6 Remarks on Truncation and Reduced-Order Modeling 7-28
7.7 Application of a Continuation Method 7-29
7.8 Recirculation Zones, Hysteresis and Control of Combustion Instabilities 7-33
7.9 Representing Noise in Analysis of Combustor Dynamics 7-37
7.10 System Identification for Combustor Dynamics with Noise 7-40
7.11 Pulsed Instabilities; Subcritical Bifurcations 7-44
7.11.1 Pulsing Solid Propellant Rockets 7-45
7.12 Dependence of Wall Heat Transfer on the Amplitude of Oscillations 7-52
7.13 One Way to Analyze the Behavior of Pulses 7-54
7.13.1 Some Results for the Propagation of Pulses 7-57
7.14 Spatial Averaging and Dynamical Systems Theory Applied to Pulsed Instabilities 7-60

Chapter 8 – Passive Control of Combustion Instabilities **8-1**
8.1 Interpretation of Passive Control 8-1
8.2 Baffles 8-4
8.3 Resonators and Acoustic Liners 8-12
8.4 Damping Due to the Formation and Presence of Particulate Material 8-15
8.5 Dynamics of Injection Systems 8-19
8.6 Passive Control of Vortex Shedding 8-19

Chapter 9 – Feedback Control of Unsteady Motions in Combustors **9-1**
9.1 The Idea of Feedback Control First Applied to Combustion Systems 9-6
9.2 Early Laboratory Demonstrations 9-7
9.3 Modal Control for Combustion Systems 9-21
9.3.1 State Feedback Control Based on Spatial Averaging 9-24
9.3.2 Application to Combustion Systems 9-27
9.4 An Example of Practical Application of Feedback Control 9-30
9.5 Briefly on Progress in Practical Feedback Control of Combustor Dynamics 9-37

Chapter 10 – References **10-1**

Annex A – Equations of Motion **A-1**
A.1 General Equations of Motion A-1
A.2 Mass, Momentum and Energy Transfer by Diffusion A-3
A.3 Construction of the Single Fluid Model A-5
A.3.1 Equation for the Density A-6
A.3.2 Equation for Momentum A-7
A.3.3 Equation for Energy A-7
A.3.4 Equation for the Pressure A-10
A.4 Muster of Equations A-10
A.5 Expansions in Mean and Fluctuating Variables A-12
A.6 Definitions of the Ordering Brackets A-13

Annex B – The Equations for One-Dimensional Unsteady Motions **B-1**
B.1 Equations for Unsteady One-Dimensional Motions B-3

Annex C – The Acoustic Boundary Layer **C-1**

Annex D – Accounting for Waves of Vorticity and Entropy **D-1**

Annex E – Accommodating Discontinuities in the Method of Spatial Averaging **E-1**

Annex F – Basis Functions Satisfying Homogeneous Boundary Conditions **F-1**

Annex G – Nyquist Criterion **G-1**
G.1 The Criterion Stated Without Proof G-2
G.2 Some General Properties of Polar Plots G-3
G.3 Construction of Nyquist Contours (Polar Plots) G-4
G.4 Definition of Encirclement G-7
G.5 Proof of Nyquist's Criterion G-8
G.5.1 Principle of the Argument G-8
G.5.2 Proof of Nyquist's Criterion G-10
G.6 Examples of Nyquist's Criterion G-13
G.7 Relative Stability; Gain and Phase Margins G-15

Annex H – Some Alternative Methods of Approximately Analyzing Nonlinear Behavior **H-1**

List of Figures

Figure		Page
Figure 1.1	Schematic diagram of a combustion system as a feedback amplifier	1-4
Figure 1.2	A chronology of combustion instabilities	1-6
Figure 1.3	Pulses of the injected fuel stream in the F-1 engine	1-8
Figure 1.4	The F-1 engine and the face of the injector showing the fourteen baffles	1-8
Figure 1.5	Injection elements commonly used in liquid rockets	1-9
Figure 1.6	Resurging subsequent to a bomb-induced perturbation	1-10
Figure 1.7	Decaying pressure induced initially by a pulse from a 13.5 grain bomb	1-10
Figure 1.8	Simplified forms of coaxial injectors	1-11
Figure 1.9	Pintle injector in the lunar excursion module (LEM) descent engine of the Apollo vehicle	1-11
Figure 1.10	Cutaway display model of the mixing head of the TRW pintle engine	1-11
Figure 1.11	Decay of pulses injected in the TRW LEM engine	1-12
Figure 1.12	Cutaway drawing of the main injector assembly of the Space Shuttle main engine	1-12
Figure 1.13	The RD-0110 engine	1-13
Figure 1.14	Four RD-0110 engines mounted in the upper stage of the Soyuz vehicle	1-14
Figure 1.15	RD-0110 engine cutaway to show combustible ribs intended to attenuate a tangential mode	1-14
Figure 1.16	Stylized configuration of a liquid-fueled ramjet	1-15
Figure 1.17	The Caltech dump combustor	1-16
Figure 1.18	Waveform and spectrum for an instability in the Caltech dump combustor	1-16
Figure 1.19	Flight test record of pressure in a Minuteman II, stage 3	1-18
Figure 1.20	Frequencies and amplitudes of combustion instabilities in the Minuteman II, stage 3 motor	1-19
Figure 1.21	A sketch of the basic T-burner and two variants	1-20
Figure 1.22	A partial summary of U.S. solid propellant motors (1951–1997) having problems of combustion instabilities	1-21
Figure 1.23	An approximate schematic summary of the important rate processes in a liquid rocket combustor	1-23
Figure 1.24	A simplified diagram for the dynamics of a liquid rocket engine	1-23
Figure 1.25	A sketch showing some of the processes taking place in a coaxial injector	1-24
Figure 1.26	Generation of an acoustic field by vortex shedding	1-26
Figure 1.27	A form of axisymmetric lean premixed combustor	1-28
Figure 1.28	A typical 2-D dump combustor	1-28
Figure 1.29	Transient growths and “limit cycles” of combustion instabilities	1-30
Figure 1.30	Growth of a nonlinear instability in a solid rocket showing the unsteady pressure	1-30

Figure 1.31	A waterfall plot taken during a stable firing	1-31
Figure 1.32	A waterfall plot showing an instability, having fundamental frequency < 200 Hz but with developing components at higher frequencies	1-31
Figure 1.33	A severe pulsed instability in a solid rocket; an example of a subcritical bifurcation	1-32
Figure 1.34	Transient behavior of (a) self-excited linearly unstable motions; (b) forced motions	1-34
Figure 1.35	Qualitative dependence of (a) energy gains and losses; and (b) the frequency response of a combustor	1-36
Figure 1.36	Exponential growth and development of a limit cycle out of a linearly unstable motion	1-38
Figure 1.37	Stability boundaries for a laboratory gas-fueled rocket	1-39
Figure 1.38	Predicted stability boundary for a large solid propellant rocket motor, and the separate contributions to α_d and α_g	1-40
Figure 1.39	A computed limit cycle and its normalized spectrum executed by a single nonlinear acoustic mode in the presence of noise	1-42
Figure 1.40	An example of fitting T-burner data with the model of a simple nonlinear oscillator	1-46
Figure 1.41	Steep-fronted waves observed in a liquid propellant rocket motor	1-47
Figure 1.42	Steep-fronted waves observed in solid propellant rocket motors	1-48
Figure 1.43	A comparison of observed and simulated steep-fronted waves in a solid propellant rocket motor	1-48
Figure 2.1	The four possible mechanisms for combustion instabilities in solid propellant rockets	2-2
Figure 2.2	Sketch of steady combustion of a solid propellant	2-3
Figure 2.3	View of the surface of a burning solid propellant containing aluminum	2-4
Figure 2.4	Representation of the temperature field for a burning solid propellant	2-6
Figure 2.5	Measurements showing the presence of the lowest tangential mode during ‘sonant’ burning	2-8
Figure 2.6	Mean pressure in a cylindrical motor operating with double-bore propellant	2-10
Figure 2.7	Sketch of the first test device having the form of a T-burner	2-11
Figure 2.8	Adapted from Hart and McClure (1959). Part (a): a sketch of their model of surface combustion; Part (b) a result for the quantity proportional to the real part of the admittance function	2-12
Figure 2.9	The first reported values of the real and imaginary parts of the admittance function for a burning surface	2-17
Figure 2.10	A summary of the instability problems for solid propellant rockets in 1970	2-18
Figure 2.11	A view of the areas of research and their connections in solid propellant rockets	2-20
Figure 2.12	Reference system and matching conditions for the QSHOD Model	2-22
Figure 2.13	Sketch of the model of a solid propellant burning with uniform combustion in the gas phase	2-25
Figure 2.14	Real and imaginary parts of a QSHOD response function computed with equation 2.67: $n_s = 0$, $A = 6.0$, $B = 0.60$	2-29
Figure 2.15	Experimental results for the change of burning rate of two double-base propellants	2-31
Figure 2.16	The real part of the response function vs. the non-dimensional frequency, $a_t w/r^2$ for A-13 propellant: the solid curve is calculated from the QSHOD formula for the values of A and B shown	2-31

Figure 2.17	The real part of the response function vs. the non-dimensional frequency for A-35 propellant	2-32
Figure 2.18	Real part of the response function measured with T-burners	2-32
Figure 2.19	Spatial definition of the model	2-38
Figure 2.20	QSHOD response function with a time lag	2-39
Figure 2.21	Combustion response, QSHOD model with gas phase dynamics	2-40
Figure 2.22	Effects of activation energy and density on the dynamics of the surface layer	2-41
Figure 2.23	Combustion response function including the dynamics of the surface layer and the gas phase	2-41
Figure 2.24	Results of a simulation with a QSHOD combustion response	2-43
Figure 2.25	Results of a simulation including dynamics of a surface layer	2-43
Figure 2.26	Results of a simulation including a time delay ($\tau = 1.5$)	2-43
Figure 2.27	Results of a simulation including dynamics of a surface layer and the gas phase	2-43
Figure 2.28	Sketches illustrating two primary processes involved in the generation of vorticity	2-45
Figure 2.29	Sketch of an apparatus for demonstrating the excitation of acoustic modes by vortex shedding at a pair of annuli	2-47
Figure 2.30	Vortex shedding from (a) an annular obstacle composed of residual inhibitor material; and (b) an edge of a backward facing step at a transition zone; (c) due to instability of the mean flow near a burning surface (parietal vortex shedding)	2-48
Figure 2.31	Typical flow between the baffles when a pure tone is generated	2-49
Figure 2.32	Experimental results for the excitation of acoustic modes by vortex shedding	2-50
Figure 2.33	Vaporization response according to Heidmann and Wieber	2-54
Figure 2.34	Graphical definition of a simple time lag	2-57
Figure 2.35	Early measurements of the time lag and pressure index in a gas rocket	2-59
Figure 2.36	Sketch of a simple case with a single entropy wave *(S)* and acoustic pressure waves (P_+, P_-)	2-60
Figure 2.37	Large-scale coherent structures in a mixing layer at 8 atmospheres in flow past a splitter plate	2-62
Figure 2.38	Photograph of vortices shed in a reacting flow	2-63
Figure 2.39	Flow past a bluff body flameholder under two conditions of flow at approximately the same speed	2-64
Figure 2.40	Development of a vortex during one cycle of a pressure oscillation	2-66
Figure 2.41	Development of a vortex at the interface of two unlike fluids	2-67
Figure 2.42	An elementary model of combustion in vortices as a mechanism for driving acoustic waves	2-67
Figure 2.43	Vortex shedding from a backward-facing step, followed by impingement on an obstacle	2-71
Figure 2.44	Normalized spectra of pressure fluctuations at the step of Figure 2.43 for two mean flow velocities at the dump plane	2-72
Figure 2.45	Coupling between pressure oscillations, structural vibrations and the injection system	2-73
Figure 2.46	Some results of calculations based on the n- τ model	2-75

Figure 2.47	Comparison of calculations and some experimental results interpreted with the n- τ model	2-75
Figure 2.48	A general representation of stability based on the $n - \tau$ model	2-76
Figure 2.49	Stability boundaries inferred with the $n - \tau$ model applied to a gas-fueled rocket	2-77
Figure 2.50	Experimental results for n and τ, using stable hypergolic propellants and various injectors	2-78
Figure 2.51	Two examples of aircraft gas turbine engines with thrust augmentation	2-80
Figure 2.52	Schematic flight envelope showing typical regions of high- and low-frequency instabilities in thrust augmentors	2-82
Figure 2.53	Sketches of the evolution of the geometry of gas turbine engines	2-82
Figure 2.54	Pratt and Whitney F100-PW-100 Augmented Turbofan Engine	2-83
Figure 2.55	Spectra of pressure oscillations observed in thrust augmentors	2-83
Figure 2.56	Flow in the vicinity of typical flameholders emphasizing the recirculation zones	2-84
Figure 2.57	A flame stabilized on a gutter in a tube	2-85
Figure 2.58	Two simple ramjet configurations using stabilization at abrupt changes of area	2-86
Figure 2.59	Model used for measurements in the inlet to a dump combustor	2-88
Figure 2.60	Comparison of measured and calculated mode shapes in the inlet of a laboratory dump combustor	2-89
Figure 2.61	Simple apparatus for demonstrating a singing flame	2-91
Figure 2.62	Location of the flame shown in Figure 2.61 to maintain a continuous tone, i.e. 'singing'	2-92
Figure 2.63	Concentrations at equilibrium of carbon monoxide and oxides of nitrogen	2-93
Figure 2.64	General behavior as the equivalence ratio is reduced near the lean blowout limit	2-94
Figure 2.65	Sketch of an electrically heated Rijke tube	2-95
Figure 2.66	The effect of tube length and heater location on the generation of oscillations in a Rijke tube	2-97
Figure 2.67	The effect of a second heater on the stability of oscillations	2-98
Figure 2.68	A prediction of a stability limit	2-99
Figure 2.69	A sketch of the horizontal Rijke tube used by Matveev	2-100
Figure 2.70	Comparison of experimental data and calculations for stability boundaries of the Rijke tube	2-101
Figure 2.71	Region of integration for obtaining matching conditions	2-102
Figure 2.72	(a) Illustrating the independence of heat transfer on flow direction; (b) illustrating possible dependence of heat transfer on flow direction due to microscopic properties of the surface	2-111
Figure 2.73	Three sorts of nonlinear behavior due to rectification distinguished by the relative values of the mean and fluctuating components of velocity	2-112
Figure 2.74	A basic flame-driven Rijke tube	2-112
Figure 5.1	Wave motions subsequent to an initial pressure pulse at rest	5-7
Figure 5.2	Reflection of a pressure pulse from a rigid surface	5-7
Figure 5.3	Comparison of spherical and planar waves produced by regions of increased pressure	5-9

Figure 5.4	Generation of spherical, planar and cylindrical waves by point sources	5-10
Figure 5.5	Acoustic energy and intensity	5-14
Figure 5.6	A graph of decibels versus pressure (lbs./in^2)	5-15
Figure 5.7	A graph showing the audible range of hearing for a typical human subject	5-15
Figure 5.8	Reflection of a plane wave	5-19
Figure 5.9	A uniform tube having a single discontinuity	5-22
Figure 5.10	Comparison of experimental and theoretical results for normal frequencies in a T-burner at ambient temperature	5-23
Figure 5.11	Rectangular chamber	5-25
Figure 5.12	Circular cylindrical coordinates	5-26
Figure 5.13	The first six transverse modes in a circular cylinder	5-26
Figure 5.14	Acoustic velocity and temperature distributions for the fundamental mode in a closed tube	5-27
Figure 5.15	Definition of the lateral control surface	5-32
Figure 5.16	The basic impedance tube	5-38
Figure 5.17	Sketch of $\lvert\hat{p}\rvert$ when distributed losses are ignored	5-39
Figure 6.1	Basic physical model of a liquid rocket used in the Princeton theoretical work	6-3
Figure 6.2	L^* versus chamber pressure, showing the stability limit for oscillations	6-6
Figure 6.3	Block diagram of the system, adapted from Figure 1 of Coates, Cohen, and Harvill	6-6
Figure 6.4	L^* instability data for two propellants	6-7
Figure 6.5	Two examples of natural cessation of L^* instabilities	6-8
Figure 6.6	Five pressure records of chuffing	6-8
Figure 6.7	Real and Imaginary parts of a simple response function	6-11
Figure 6.8	The L^*-chart for determining the parameters A and B	6-13
Figure 6.9	A tube with distributed heat addition and an oscillating piston to drive waves	6-17
Figure 6.10	The Caltech dump combustor	6-20
Figure 6.11	Experimental confirmation of Rayleigh's Criterion	6-21
Figure 6.12	Geometrical parameters defined by Brownlee and a typical time history of the mean chamber pressure	6-29
Figure 6.13	Stability boundaries for the 010 mode: experimental and theoretical	6-30
Figure 6.14	Dependence of the admittance function on frequency inferred from Brownlee's measurements	6-30
Figure 6.15	Velocity vectors for flow through a permeable boundary	6-36
Figure 6.16	The origin of 'flow-turning': An element of fluid shown (a) at entry normal to a burning or porous surface, and (b) in an intermediate state of the process of acquiring the local axial velocity having mean and acoustic components	6-37
Figure 6.17	Grains used in laboratory motors for which data were taken for unstable oscillations	6-43
Figure 6.18	A comparison of data taken with the motors shown in Figure 6.17 with predictions according to (6.161)	6-43
Figure 6.19	Calculation of the decay of the fundamental mode following a pulse	6-43

Figure 6.20	A *T*-burner chart for determining the parameters *A* and *B*	6-46
Figure 6.21	Growth of unstable motions according to the approximate analysis; and a numerical simulation of the same problem	6-49
Figure 6.22	Spectrum for the calculated waveform shown in Figure 6.21(b)	6-51
Figure 6.23	Effect of truncation in the waveforms	6-52
Figure 6.24	Stylized rendition of azimuthal and streamwise vortices in a solid propellant rocket chamber	6-53
Figure 6.25	Some configurations used in observations of vortex shedding and instabilities	6-54
Figure 6.26	The five two-dimensional configurations of the LP3 test motors	6-55
Figure 6.27	Upper image: flow visualization of parietal vortex shedding, using PLIF with acetone; Lower image: result of numerical calculations	6-56
Figure 6.28	The three kinds of vortex shedding	6-56
Figure 6.29	A sketch of the LP6 test device	6-56
Figure 6.30	Experimental results for a test in the LP6 configuration showing parietal vortex shedding	6-57
Figure 6.31	Idealized axisymmetric and two-dimensional nozzles for calculating admittance functions	6-60
Figure 6.32	Numerical values for the admittance function of a nozzle exposed to isentropic oscillations	6-63
Figure 6.33	Results for the real part of the admittance function for a nozzle	6-63
Figure 6.34	Results for the imaginary part of the admittance function for a nozzle	6-63
Figure 6.35	Comparison of the theoretical and experimental admittance results for nozzle 15-08-2.5, longitudinal modes	6-64
Figure 6.36	Comparison of the theoretical and experimental admittance results for nozzle 45-08-2.5, longitudinal modes	6-65
Figure 6.37	Comparison of the theoretical and experimental admittance results for nozzle 45-16-2.5, longitudinal modes	6-65
Figure 6.38	Attenuation of acoustic waves caused by small particles suspended in a gas	6-69
Figure 6.39	Frequency shift of acoustic oscillation caused by small particles suspended in a gas	6-69
Figure 6.40	Mass distribution of subfractions of the combustion residue of NWC Mix No. 10	6-69
Figure 6.41	A possible explanation of increased growth rate due to distributed combustion of aluminum in a T-burner	6-73
Figure 6.42	General view of the Ariane 5 strap-on booster motor P230	6-74
Figure 6.43	Results for the unsteady pressure measured in a static test firing of the Ariane 5, P230 motor	6-75
Figure 6.44	Shape of the grain used in simulations of two-phase unsteady flow in the Ariane 5, P230 motor	6-76
Figure 7.1	Steepening of a planar compressive wave into a shock wave	7-3
Figure 7.2	Acoustic streaming, a consequence of small viscous effects and nonlinear acoustics	7-4
Figure 7.3	Finite streaming motions in a motor having five slots, giving net counterclockwise circulation	7-5
Figure 7.4	An oscillating shock wave in a closed tube	7-5

Figure 7.5	Closed tube containing heater at its midpoint	7-6
Figure 7.6	Model of a liquid rocket analyzed in the Princeton works	7-7
Figure 7.7	Waveforms observed in a gas rocket	7-8
Figure 7.8	Steepening of a longitudinal wave travelling in a cylindrical chamber; $\lvert p'/p \rvert$ vs. x	7-10
Figure 7.9	The envelope of oscillations for the T-burner firing shown in Figure 1.36	7-11
Figure 7.10	Data for the growth and decay periods of the firing shown in Figure 7.9	7-13
Figure 7.11	Nonlinear growth of oscillations for several values of the initial amplitude ratio $A_0=A_m$	7-14
Figure 7.12	Nonlinear decay of oscillations for several values of the initial amplitude ratio $A_0=A_m$	7-14
Figure 7.13	A T-burner firing showing stronger nonlinear behavior during the initial portion of the decay of oscillations	7-15
Figure 7.14	The two cases presented by equation (7.22)	7-16
Figure 7.15	The two cases presented by equation (7.24). Part (b) represents a pulsed (triggered) instability	7-17
Figure 7.16	Energy flow in the two-mode approximation	7-18
Figure 7.17	Regions of stability for two longitudinal modes, time-averaged equations	7-21
Figure 7.18	Effects of truncation for a stable limit cycle: first mode unstable, second mode stable	7-21
Figure 7.19	Effects of truncation for an unstable limit cycle: first mode unstable, second mode stable	7-22
Figure 7.20	Development of a stable limit cycle when the first mode is stable but the second mode is unstable	7-22
Figure 7.21	Amplitudes in two limit cycles involving the first tangential and first radial modes only	7-26
Figure 7.22	An example of limit cycles for three normal modes, 1T/1R/2T	7-26
Figure 7.23	Sketch of the interpretation as a ‘detonation’ wavefront	7-28
Figure 7.24	The test engine, 20,000 lbf-thrust engine and components	7-28
Figure 7.25	Schematic illustration of the continuation method applied to limit cycles	7-29
Figure 7.26	Two examples of bifurcation	7-30
Figure 7.27	The effects of truncation for a two-mode system: second mode unstable, all other modes stable	7-31
Figure 7.28	Effect of time-averaging for two modes	7-31
Figure 7.29	Solutions with truncation of the time-averaged equations	7-32
Figure 7.30	Maximum amplitude of η_1 in the limit cycle; four modes, comparison of results for the full oscillator and the time-averaged equations	7-32
Figure 7.31	Hysteresis loop for a recirculated zone idealized as a simple chemical reactor	7-34
Figure 7.32	Sketch of a recirculation zone formed by a jet of fuel or oxidizer	7-34
Figure 7.33	Stability boundary and an idealized hysteresis loop for the Caltech dump combustor	7-34
Figure 7.34	(a) Sketch of the dump combustor modified to allow injection of pulses; (b) hysteresis observed in the apparatus shown in (a)	7-35
Figure 7.35	Pressure traces and spectra for the two branches of the hysteresis loop	7-35

Figure 7.36	Time history of a portion of trace	7-36
Figure 7.37	Shadowgraphs of the transition induced by a single pulse initiated at $t = 0$	7-37
Figure 7.38	Pressure trace and spectrum for a simulation with noise; four modes, first mode unstable	7-40
Figure 7.39	Simulated pressure trace with noise; all modes stable	7-42
Figure 7.40	Application of Berg's method: power spectrum of the pressure trace in Figure 7.39 and its reconstruction	7-42
Figure 7.41	Dependence of the peak amplitudes of the power spectra for four modes, on noise power	7-43
Figure 7.42	Reconstructed pressure trace for the transient response excited by a 10% pulse	7-43
Figure 7.43	Values of decay rates (modal attenuation) found with Berg's method with multiplicative (ξ_n) noise	7-44
Figure 7.44	Values of decay rates (modal attenuation) found with the method of pulsing	7-44
Figure 7.45	A pulsed instability initiated in an 80 inch long motor by a small explosive charge	7-46
Figure 7.46	Picture taken with schlieren apparatus midway between the head-end and the nozzle	7-46
Figure 7.47	Stability boundaries for three propellants tested in 80 inch motors and the influence of mean pressure (throat diameter) on stability	7-47
Figure 7.48	Stability boundary for 40 inch rockets, 5 inch diameter showing chamber pressure and burning rate below which pulses of pressure are stable	7-48
Figure 7.49	Response of the head end of a motor (L = 10 ft.) to a continuously applied input pressure	7-49
Figure 7.50	Three types of pulsing devices used by Baum, Lovine and Levine	7-50
Figure 7.51	Time evolution of pressure perturbations produced by a pyro pulser	7-50
Figure 7.52	Comparison of measured and predicted pressures at the head end, test PCC4	7-51
Figure 7.53	Response functions for the three propellants used by Baum, Levine and Lovine	7-51
Figure 7.54	Comparison of observed and calculated pulsed instabilities in an aft-finocyl motor	7-51
Figure 7.55	Mean heat transfer coefficient measured in T-burners showing dependence on amplitude of pressure amplitude	7-53
Figure 7.56	Mean heat transfer rates correlated as mean Nusselt number versus mean Reynolds number	7-53
Figure 7.57	Superposition of two pulse trains to represent propagation of a single pulse in a closed tube	7-55
Figure 7.58	Propagation of pulses developed from a stationary initial pulse in a closed tube	7-55
Figure 7.59	Approximations to a rectangular initial pulse by 10, 30, 50 modes	7-58
Figure 7.60	Propagation of a rectangular pulse without losses (10 modes)	7-59
Figure 7.61	Propagation of an evolving cosine wave	7-59
Figure 7.62	Propagation of a rectangular pulse (10 modes)	7-59
Figure 7.63	The function $F(u')$ used in the model having a threshold velocity $u_t > 0$; the value $u_t = 0.02$ is only an example	7-61
Figure 7.64	Maximum amplitude of the first acoustic mode in the limit cycle showing the contribution of nonlinear gasdynamics and combustion	7-62

Figure 7.65	Maximum amplitude of the first acoustic mode in the limit cycle showing a subcritical bifurcation with a turning point only if both nonlinear gasdynamics and nonlinear combustion dynamics are accounted for	7-62
Figure 8.1	Schematic diagram showing qualitatively the conditions under which instabilities exist	8-2
Figure 8.2	Qualitative and stylized interpretation of the possible effect of modifications on the dynamical behavior of a combustion system	8-2
Figure 8.3	Block diagram of the basic system (G,Q) with passive control due to modifications of the chamber dynamics (C_G) and of the combustion dynamics (C_Q)	8-3
Figure 8.4	Simple forms of baffles for solid propellant rockets	8-5
Figure 8.5	Early examples showing consequences of high-frequency oscillations	8-6
Figure 8.6	The importance of baffles in the F-1 engine	8-6
Figure 8.7	Perspective view of a rocket combustion chamber with longitudinal baffles installed on the injector face	8-7
Figure 8.8	An injector with baffles and azimuthal slots	8-7
Figure 8.9	Examples of the arrangement and shapes of baffles fixed to the injector face of a liquid rocket	8-8
Figure 8.10	The pressure fields for the lowest order modes commonly encountered in circular cylindrical combustion chambers	8-8
Figure 8.11	Combustible longitudinal baffles (ribs) in the Russian RD-0110 engine and their substantial effects on the amplitude of oscillations	8-10
Figure 8.12	The combustion chamber with a three-bladed baffle analyzed by Wicker, Yoon and Yang	8-10
Figure 8.13	A model of a rocket chamber and the measured spectrum of tangential modes	8-11
Figure 8.14	Two methods for measuring the damping factor: (a) decay rate, α; (b) bandwidth	8-11
Figure 8.15	Results for the decay rate of tangential modes in the chamber shown in Figure 8.13(a)	8-11
Figure 8.16	The essential features of an idealized cavity resonator	8-12
Figure 8.17	Convectively cooled thrust chamber fitted with an acoustic liner	8-14
Figure 8.18	Apparatus for measurement of the acoustic properties of a model of a liner with bias flow	8-14
Figure 8.19	Absorption coefficient for a portion of acoustic liner	8-15
Figure 8.20	Scheme of a procedure followed to improve the attenuation of combustion instabilities by increasing the particle damping	8-17
Figure 8.21	Particle size distribution for tactical motor propellant and a conventional aluminized propellant	8-18
Figure 8.22	Comparison of particle size distribution to optimum size for damping in motor	8-18
Figure 8.23	Flow past a bluff body flameholder under two conditions of flow at approximately the same speed	8-19
Figure 8.24	Vortex shedding from a rearward-facing step	8-20
Figure 8.25	Examples of configurations used to study vortex shedding in combustion chambers	8-21
Figure 8.26	Sketch of the origin and elimination of pressure oscillations produced in the chamber by vortex shedding in a solid propellant rocket	8-22

Figure 8.27	'Waterfall' plots of low-frequency oscillations observed in a Minuteman III Stage 3 motor	8-22
Figure 8.28	Cylindrical jet of carbon monoxide into air, initial Reynolds number approximately 30,000 (average speed, 127 ft/s)	8-23
Figure 8.29	Test equipment for studying reacting and non-reacting jet flows in a dump combustor	8-25
Figure 8.30	An example of test results showing the presence of coherent vortices when the acoustic forcing frequency f_F equals the preferred mode frequency f_{pm}, the frequency at the maximum of the response	8-26
Figure 9.1	Block diagram for a system containing passive (C_G and C_Q) and feedback (C_f and C_{fb}) control	9-3
Figure 9.2	General block diagram of a combustion system with passive and active control	9-4
Figure 9.3	A general block diagram for classical and modern control	9-5
Figure 9.4	Schematic of the first proposal for active feedback control of the dynamics in a combustion system	9-6
Figure 9.5	Block diagram for the system shown in Figure 9.4	9-6
Figure 9.6	Feedback control of a Rijke tube by injection of acoustic waves	9-8
Figure 9.7	An early example of control based on Rayleigh's Criterion	9-8
Figure 9.8	The geometry of the rig at Rolls-Royce, Derby, with one set of flow conditions	9-10
Figure 9.9	Cambridge apparatus	9-11
Figure 9.10	Magnitude and phase of the pressure distribution and the transfer function for heat release for one configuration of the test apparatus shown in Figure 9.9(a)	9-12
Figure 9.11	Unsteady motions in the test rig shown in Figure 9.9(a)	9-14
Figure 9.12	The device for mixing secondary fuel and air at the upstream end of the flameholder shown in Figure 9.9	9-14
Figure 9.13	Apparatus used in the first tests of feedback control of combustion at École Centrale and the Technische Universität München. (a) Tubular burner exhibiting oscillations; (b) the configuration with provision for feedback control	9-15
Figure 9.14	Feedback control at École Centrale	9-17
Figure 9.15	GE demonstration of feedback control	9-18
Figure 9.16	A Nyquist diagram recorded by Gulati and Mani for their combustor with their first controller	9-19
Figure 9.17	Schematic of a Rijke tube apparatus (a) and the block diagram (b) for loop shaping design	9-20
Figure 9.18	Open and closed loop power spectra for the Rijke tube shown in Figure 9.17(a)	9-21
Figure 9.19	The general scheme according to the procedures followed here for connecting the physical system (a combustor), physical modeling, mathematical modeling, dynamics and control	9-22
Figure 9.20	A simple block diagram for proportional-integral-derivative (PID)	9-28
Figure 9.21	Block diagram for PID control applied to a linear system; the transfer function *G(s)* is given by equation Figure 9.52	9-29
Figure 9.22	Simplified sketch of the system analyzed by Fung, Sinha, and Yang	9-29
Figure 9.23	The apparatus and stability diagram for the results with no feedback control reported by Schimmer and Vortmeyer	9-31

Figure 9.24	Feedback control at Technische Universität München	9-32
Figure 9.25	Test results found with the apparatus shown in Figure 9.24	9-33
Figure 9.26	Test apparatus and feedback control with a simple liquid fuel nozzle and the direct drive valve (DDV)	9-34
Figure 9.27	A sketch of a combustor used in the Vx4.3A series Siemens gas turbines and an illustration of the size of the combustor	9-35
Figure 9.28	Simplified sketches of the Siemens Hybrid Burner	9-35
Figure 9.29	Minimum number of four actuators, one sensor and one controller to control a standing second azimuthal mode	9-36
Figure 9.30	Addition of the cylindrical burner outlet (CBO) to the Siemens Hybrid Burner	9-36
Figure C.1	Sketch of the flow in an acoustic boundary layer showing the velocity field at two instants separated by one-half period	C-1
Figure E.1	Three examples of discontinuities in a flow field	E-2
Figure E.2	A discontinuity in a one-dimensional flow	E-2
Figure F.1	The problem defined for determining $p(x, y)$	F-2
Figure G.1	The system $G(s)$ with controller $H(s)$ and negative unity feedback	G-1
Figure G.2	Definition of the Nyquist path excluding poles of HG on the imaginary axis. Examples of poles and zeros within the path are not excluded	G-2
Figure G.3	Indentations around poles on the imaginary axis. Poles and zeros within the contour are supressed	G-4
Figure G.4	Definitions of positive and negative encirclements	G-8
Figure G.5	Illustrating no encirclement ($N_0 = 0$) and two negative encirclements ($N_0 = -2$) by $f(s)$ when s executes a closed contour encircling the origin	G-9
Figure G.6	Examples showing consistent maintenance of the definition of 'interior' of a path under mapping from the s-plane to the G-plane	G-10
Figure G.7	An example showing two negative encirclements of the origin by the path obtained by mapping the Nyquist contour	G-11
Figure G.8	Mappings of the Nyquist contour in the HG and $1 + HG$ planes	G-11
Figure G.9	Examples of positive and negative encirclements of –1 by mappings of the Nyquist contour	G-12
Figure G.10	Forming the Nyquist plot for $HG = \frac{K}{s(s+p_1)(s+p_2)}$	G-14
Figure G.11	A stable second order system with an integrator	G-15
Figure G.12	The Nyquist plot for the system shown in Figure G.11	G-16
Figure G.13	The Nyquist plot for 'any' HG near $\|HG\|_\pi$, i.e., the magnitude of HG when the phase or arg of HG is π	G-16
Figure G.14	The Bode plot for the system shown in Figure G.11, $K = 1$	G-18

List of Tables

Table		Page
Table 2.1	Typical Values of Combustion and Physical Properties	2-21
Table 2.2	Principal Characteristics of the MURI Propellants	2-33
Table 5.1	Results for Rightward and Leftward Traveling Sinusoidal Waves	5-16
Table 5.2	Some Formulas for Incident and Reflected Plane Waves	5-19
Table 6.1	Values of the Geometrical, Combustion and Physical Properties	6-50
Table 6.2	Linear Growth Constants and Frequency Shifts	6-51
Table 6.3	Total Values of the Linear Growth Constants and Frequency Shifts	6-51
Table 6.4	Frequencies and Amplitudes of Acoustic Pressures	6-51
Table 6.5	Mass Fractions of NWC Mix No. 10 Residue for Particles Smaller than 43 μm	6-70
Table 6.6	Parameters Used in the Particle Damping Calculations	6-70
Table 6.7	Damping Coefficients Measured, NWC Mix No. 10, for Particle Sizes Smaller than μm	6-71
Table 6.8	Measured and Calculated Damping Coefficients for NWC Mix and 10	6-71
Table 7.1	Summary of Some Works from Princeton on a Theory of Combustion Instabilities	7-7
Table 7.2	Values of the Linear Parameters	7-41
Table 7.3	Relation Between the Noise Power of Ξ_n and the rms Value of the Simulated Pressure Fluctuation	7-41
Table 8.1	Considerations of Baffles for Application to Solid Propellant Rockets	8-9
Table D.1	Definition of Splitting a Disturbance in the Three Modes of Propagation	D-4
Table D.2	Zeroth Approximation for the Flow Variables in the Three Modes	D-4

Foreword

Propulsion systems, which are one of the priority activities of the Applied Vehicle Technology Panel of NATO's Research and Technology Organisation, are frequently confronted by unexpected and unsteady behaviors known by the generic name of "combustion instabilities". Many solid propellant rockets, liquid propellant engines, ramjets and main or reheat combustors of turbojets have been affected by these types of problems during development. Combustion instabilities were identified at the start of the 1950s as an endemic disease and were then the subject of research aimed at understanding their origins, explaining how they developed and, eventually, predicting their levels. The researchers were very quickly convinced of the difficulty of the problem, which is essentially due to two factors: firstly, the difficulty of taking detailed measurements of the internal flow in engines, because of the extremely severe physical conditions inside them, and secondly, the close coupling between numerous unsteady mechanisms related to fluid mechanics, combustion, two-phase flows, etc. The work done on this subject in the United States has had a profound influence in all Western countries and I had the good fortune, when I was asked to study the question for the French Armament Procurement Agency (DGA), to meet Professors Fred Culick and Ed Price, then later Professor Gary Flandro and other US Navy, US Air Force and NASA specialists. These contacts were determining factors for the direction of French work.

Today, Professor Fred Culick proposes a summary entitled "Unsteady Motion in Combustors for Propulsion Systems" in the form of an AGARDograph. There are very few scientists in the world who have accumulated such in-depth expertise and experience on the subject and the RTO should be grateful to Professor Fred Culick for having put all this acquired knowledge at the service of NATO's technological research. An attentive reading of the document prepared reveals that it is a truly comprehensive survey, in the literal sense of the word. What Professor Fred Culick has done is to put several decades of research into an understandable form, thus endowing the work with a true encyclopaedic nature, both by the variety of situations examined and by the abundance and exhaustiveness of the references used. Due to his great teaching ability, Professor Fred Culick has also been able to conduct a quite weighty mathematical analysis with thoroughness and accuracy and to establish the indispensable link between observations made on engines and predictions arrived at by calculation. Furthermore, if only one of the work's qualities had to be pointed out, I, for my part, would opt for Professor Fred Culick's exceptional ability to give physical meaning to the equations.

I therefore think that the AGARDograph prepared by Professor Fred Culick is bound to become a worldwide reference on the difficult but always topical subject of combustion instabilities.

Dr. Paul KUENTZMANN
ONERA, France
Former PEP/AGARD member
Former AVT/RTO member
RTB member

Avant-propos

Les systèmes propulsifs, qui constituent l'une des priorités des activités de la Commission Applied Vehicle Technology de la Research and Technology Organisation de l'OTAN, sont fréquemment confrontés à des comportements instationnaires imprévus connus sous le nom générique « d'instabilités de combustion ». De nombreux moteurs-fusées à propergol solide, moteurs-fusées à ergols liquides, statoréacteurs, foyers principaux ou de rechauffe de turboréacteurs ont connu ce type de problème en cours de développement. Identifiées au début des années 50 comme une maladie endémique, les instabilités de combustion ont dès lors fait l'objet de recherches pour en comprendre l'origine, en expliquer le développement et, à terme, en prévoir les niveaux. Les chercheurs ont été très tôt convaincus de la difficulté du problème, qui tient pour l'essentiel à deux aspects : d'une part, à la difficulté de réaliser des mesures détaillées de l'écoulement dans les moteurs, en raison des conditions physiques très sévères qui y règnent, et, d'autre part, du fait du couplage étroit de nombreux mécanismes instationnaires relevant de la mécanique des fluides, de la combustion, des écoulements diphasiques, etc. Les travaux conduits sur ce thème aux Etats-Unis ont imprégné tous les pays occidentaux et j'ai eu la chance, lorsque j'ai été chargé d'étudier la question pour la DGA française, de rencontrer les Professeurs Fred Culick et Ed Price, puis ultérieurement le Professeur Gary Frandro et d'autres spécialistes de l'US Navy, de l'US Air Force et de la NASA. Ces contacts ont été déterminants pour orienter les travaux français.

Le Professeur Fred Culick propose aujourd'hui sous la forme d'un AGARDograph une synthèse intitulée « Unsteady Motions in Combustion Chambers for Propulsion Systems ». Il existe très peu de scientifiques au monde qui aient accumulé une expertise et un expérience aussi approfondies sur le sujet et la RTO doit être reconnaissante au Professeur Fred Culick d'avoir mis tout cet acquis au service des recherches technologiques de l'OTAN. Une lecture attentive montre que le document préparé constitue une véritable Somme, au sens littéral du mot. Le Professeur Fred Culick a en effet remis en forme plusieurs décennies de recherche, conférant ainsi à l'ouvrage un caractère véritablement encyclopédique, tant par la variété des situations examinées que par l'abondance et l'exhaustivité des références utilisées. Grâce à un sens pédagogique aigu, le Professeur Fred Culick a également su conduire, avec rigueur et précision, une analyse mathématique assez lourde, et établir la liaison indispensable entre observations réalisées sur moteurs et prévisions de calcul. Si en outre une seule qualité de l'ouvrage devait être mise en exergue, j'opterais pour ma part sur l'exceptionnelle faculté du Professeur Fred Culick à donner un sens physique aux équations.

L'AGARDograph préparé par le Professeur Fred Culick m'apparaît donc devoir devenir l'ouvrage mondial de référence sur le sujet difficile mais toujours d'actualité des instabilités de combustion.

Docteur Paul KUENTZMANN
ONERA, France
Ex membre PEP/AGARD
Ex membre AVT/RTO
Membre RTB

REPORT DOCUMENTATION PAGE

1. Recipient's Reference	2. Originator's References	3. Further Reference	4. Security Classification of Document
	RTO-AG-AVT-039 AC/323(AVT-039)TP/103	ISBNs 92-837-0059-7 978-92-837-0059-3	UNCLASSIFIED/ UNLIMITED

5. Originator

Research and Technology Organisation
North Atlantic Treaty Organisation
BP 25, F-92201 Neuilly-sur-Seine Cedex, France

6. Title

Unsteady Motions in Combustion Chambers for Propulsion Systems

7. Presented at/Sponsored by

This AGARDograph was approved by the Propulsion and Energetics Panel (PEP) of NATO's Advisory Group for Aerospace Research and Development (AGARD); subsequently it has been sponsored by the Applied Vehicle Technology Panel (AVT) of NATO's Research and Technology Organisation (RTO).

8. Author(s)/Editor(s)	9. Date
Multiple	December 2006

10. Author's/Editor's Address	11. Pages
Multiple	664

12. Distribution Statement

There are no restrictions on the distribution of this document. Information about the availability of this and other RTO unclassified publications is given on the back cover.

13. Keywords/Descriptors

Air vehicles	Decision aids	Methodology	Simulation
Computer architecture	Design	Military vehicles	Software development
Computer programs	Integrated systems	Models	Standards
Computerized simulation	Interoperability	Requirements	Systems analysis
Control theory	Land vehicles	Sea vehicles	Systems engineering
Cost effectiveness	M&S (Modelling and Simulation)		

14. Abstract

This book has several purposes, including a broad historical summary of combustion instabilities in propulsion systems; a concise compilation of the main mechanisms for instabilities in liquid and solid fueled rockets, ramjets, thrust augmentors and gas turbines; development of a theoretical framework for investigating unsteady motions in combustion systems; and accessible surveys of the basic material required to understand the subject. Emphasis is placed throughout the book on observed behavior. For well-understood reasons, the best, and in many respects most useful quantitative data, have been obtained for solid propellant rockets. Hence that type of device occupies a special position in the subject. The book comprises nine chapters and eight annexes. Material in the annexes is not essential for reading and broadly understanding the main part of the book, but is likely interesting for those choosing to do research on the subject. The nine chapters divide into three parts. Chapter 1 covers basic behavior observed in operational systems, and includes brief historical summaries as well as qualitative interpretations of observed behavior. Chapter 2 contains lengthy descriptions of a few mechanisms, especially for some of the behavior known from observations of solid propellant rockets. The second part of the book comprises Chapters 3 and 4 which summarize the development of the method of spatial averaging and construction of the systems of equations used for analytical studies in the subject; and Chapter 5, a survey of those parts of classical acoustics required to understand unsteady motions in combustors. Applications of the formal structure to problems arising in laboratory and full-scale devices are covered in the last four chapters. Chapter 6 treats linear behavior, including linear stability with examples; and Chapter 7 covers some aspects of nonlinear behavior based on the theory developed in the earlier part of the book. Practical methods of treating combustion instabilities are the subjects of the last two chapters. Chapter 8 describes various methods of passive control and how they fit in the general understanding of combustion instabilities. The last chapter is largely descriptive, a brief summary of some of the accomplishments of active control. It's an interesting and potentially very important subject which has fallen short of many optimistic expectations. There are presently no known practical applications. Successful advancement likely will rest on better understanding of fundamental behavior.

NORTH ATLANTIC TREATY ORGANISATION

BP 25
F-92201 NEUILLY-SUR-SEINE CEDEX • FRANCE
Télécopie 0(1)55.61.22.99 • E-mail mailbox@rta.nato.int

RESEARCH AND TECHNOLOGY ORGANISATION

DIFFUSION DES PUBLICATIONS RTO NON CLASSIFIEES

Les publications de l'AGARD et de la RTO peuvent parfois être obtenues auprès des centres nationaux de distribution indiqués ci-dessous. Si vous souhaitez recevoir toutes les publications de la RTO, ou simplement celles qui concernent certains Panels, vous pouvez demander d'être inclus soit à titre personnel, soit au nom de votre organisation, sur la liste d'envoi.

Les publications de la RTO et de l'AGARD sont également en vente auprès des agences de vente indiquées ci-dessous.

Les demandes de documents RTO ou AGARD doivent comporter la dénomination « RTO » ou « AGARD » selon le cas, suivi du numéro de série. Des informations analogues, telles que le titre est la date de publication sont souhaitables.

Si vous souhaitez recevoir une notification électronique de la disponibilité des rapports de la RTO au fur et à mesure de leur publication, vous pouvez consulter notre site Web (www.rta.nato.int) et vous abonner à ce service.

CENTRES DE DIFFUSION NATIONAUX

ALLEMAGNE
Streitkräfteamt / Abteilung III
Fachinformationszentrum der
Bundeswehr (FIZBw)
Gorch-Fock-Straße 7, D-53229 Bonn

BELGIQUE
Etat-Major de la Défense
Département d'Etat-Major Stratégie
ACOS-STRAT – Coord. RTO
Quartier Reine Elisabeth
Rue d'Evère, B-1140 Bruxelles

CANADA
DSIGRD2
Bibliothécaire des ressources du savoir
R et D pour la défense Canada
Ministère de la Défense nationale
305, rue Rideau, 9e étage
Ottawa, Ontario K1A 0K2

DANEMARK
Danish Defence Research Establishment
Ryvangs Allé 1, P.O. Box 2715
DK-2100 Copenhagen Ø

ESPAGNE
SDG TECEN / DGAM
C/ Arturo Soria 289
Madrid 28033

ETATS-UNIS
NASA Center for AeroSpace
Information (CASI)
Parkway Center, 7121 Standard Drive
Hanover, MD 21076-1320

FRANCE
O.N.E.R.A. (ISP)
29, Avenue de la Division Leclerc
BP 72, 92322 Châtillon Cedex

GRECE (Correspondant)
Defence Industry & Research
General Directorate
Research Directorate
Fakinos Base Camp, S.T.G. 1020
Holargos, Athens

HONGRIE
Department for Scientific Analysis
Institute of Military Technology
Ministry of Defence
P O Box 26
H-1525 Budapest

ISLANDE
Director of Aviation
c/o Flugrad
Reykjavik

ITALIE
Centro di Documentazione
Tecnico-Scientifica della Difesa
Via XX Settembre 123
00187 Roma

LUXEMBOURG
Voir Belgique

NORVEGE
Norwegian Defence Research Establishment
Attn: Biblioteket
P.O. Box 25
NO-2007 Kjeller

PAYS-BAS
Royal Netherlands Military
Academy Library
P.O. Box 90.002
4800 PA Breda

POLOGNE
Armament Policy Department
218 Niepodleglosci Av.
00-911 Warsaw

PORTUGAL
Estado Maior da Força Aérea
SDFA – Centro de Documentação
Alfragide
P-2720 Amadora

REPUBLIQUE TCHEQUE
LOM PRAHA s. p.
o. z. VTÚLaPVO
Mladoboleslavská 944
PO Box 18
197 21 Praha 9

ROUMANIE
Romanian National Distribution Centre
Armaments Department
9-11, Drumul Taberei Street
Sector 6, 77305, Bucharest

ROYAUME-UNI
Dstl Knowledge Services
Information Centre
Building 247
Dstl Porton Down
Salisbury
Wiltshire SP4 0JQ

TURQUIE
Milli Savunma Bakanlığı (MSB)
ARGE ve Teknoloji Dairesi Başkanlığı
06650 Bakanliklar – Ankara

AGENCES DE VENTE

NASA Center for AeroSpace Information (CASI)
Parkway Center, 7121 Standard Drive
Hanover, MD 21076-1320
ETATS-UNIS

The British Library Document Supply Centre
Boston Spa, Wetherby
West Yorkshire LS23 7BQ
ROYAUME-UNI

Canada Institute for Scientific and Technical Information (CISTI)
National Research Council
Acquisitions, Montreal Road, Building M-55
Ottawa K1A 0S2, CANADA

Les demandes de documents RTO ou AGARD doivent comporter la dénomination « RTO » ou « AGARD » selon le cas, suivie du numéro de série (par exemple AGARD-AG-315). Des informations analogues, telles que le titre et la date de publication sont souhaitables. Des références bibliographiques complètes ainsi que des résumés des publications RTO et AGARD figurent dans les journaux suivants :

Scientific and Technical Aerospace Reports (STAR)
STAR peut être consulté en ligne au localisateur de ressources uniformes (URL) suivant:
http://www.sti.nasa.gov/Pubs/star/Star.html
STAR est édité par CASI dans le cadre du programme NASA d'information scientifique et technique (STI)
STI Program Office, MS 157A
NASA Langley Research Center
Hampton, Virginia 23681-0001
ETATS-UNIS

Government Reports Announcements & Index (GRA&I)
publié par le National Technical Information Service
Springfield
Virginia 2216
ETATS-UNIS
(accessible également en mode interactif dans la base de données bibliographiques en ligne du NTIS, et sur CD-ROM)

NORTH ATLANTIC TREATY ORGANISATION

BP 25

F-92201 NEUILLY-SUR-SEINE CEDEX • FRANCE
Télécopie 0(1)55.61.22.99 • E-mail mailbox@rta.nato.int

RESEARCH AND TECHNOLOGY ORGANISATION

DISTRIBUTION OF UNCLASSIFIED RTO PUBLICATIONS

AGARD & RTO publications are sometimes available from the National Distribution Centres listed below. If you wish to receive all RTO reports, or just those relating to one or more specific RTO Panels, they may be willing to include you (or your Organisation) in their distribution.

RTO and AGARD reports may also be purchased from the Sales Agencies listed below.

Requests for RTO or AGARD documents should include the word 'RTO' or 'AGARD', as appropriate, followed by the serial number. Collateral information such as title and publication date is desirable.

If you wish to receive electronic notification of RTO reports as they are published, please visit our website (www.rta.nato.int) from where you can register for this service.

NATIONAL DISTRIBUTION CENTRES

BELGIUM
Etat-Major de la Défense
Département d'Etat-Major Stratégie
ACOS-STRAT – Coord. RTO
Quartier Reine Elisabeth
Rue d'Evère
B-1140 Bruxelles

CANADA
DRDKIM2
Knowledge Resources Librarian
Defence R&D Canada
Department of National Defence
305 Rideau Street, 9th Floor
Ottawa, Ontario K1A 0K2

CZECH REPUBLIC
LOM PRAHA s. p.
o. z. VTÚLaPVO
Mladoboleslavská 944
PO Box 18
197 21 Praha 9

DENMARK
Danish Defence Research Establishment
Ryvangs Allé 1
P.O. Box 2715
DK-2100 Copenhagen Ø

FRANCE
O.N.E.R.A. (ISP)
29, Avenue de la Division Leclerc
BP 72
92322 Châtillon Cedex

GERMANY
Streitkräfteamt / Abteilung III
Fachinformationszentrum der Bundeswehr (FIZBw)
Gorch-Fock-Straße 7
D-53229 Bonn

GREECE (Point of Contact)
Defence Industry & Research General Directorate
Research Directorate
Fakinos Base Camp
S.T.G. 1020
Holargos, Athens

HUNGARY
Department for Scientific Analysis
Institute of Military Technology
Ministry of Defence
P O Box 26
H-1525 Budapest

ICELAND
Director of Aviation
c/o Flugrad, Reykjavik

ITALY
Centro di Documentazione Tecnico-Scientifica della Difesa
Via XX Settembre 123
00187 Roma

LUXEMBOURG
See Belgium

NETHERLANDS
Royal Netherlands Military Academy Library
P.O. Box 90.002
4800 PA Breda

NORWAY
Norwegian Defence Research Establishment
Attn: Biblioteket
P.O. Box 25
NO-2007 Kjeller

POLAND
Armament Policy Department
218 Niepodleglosci Av.
00-911 Warsaw

PORTUGAL
Estado Maior da Força Aérea
SDFA – Centro de Documentação
Alfragide
P-2720 Amadora

ROMANIA
Romanian National Distribution Centre
Armaments Department
9-11, Drumul Taberei Street
Sector 6, 77305, Bucharest

SPAIN
SDG TECEN / DGAM
C/ Arturo Soria 289
Madrid 28033

TURKEY
Milli Savunma Bakanlığı (MSB)
ARGE ve Teknoloji Dairesi Başkanlığı
06650 Bakanliklar – Ankara

UNITED KINGDOM
Dstl Knowledge Services
Information Centre
Building 247
Dstl Porton Down
Salisbury, Wiltshire SP4 0JQ

UNITED STATES
NASA Center for AeroSpace Information (CASI)
Parkway Center
7121 Standard Drive
Hanover, MD 21076-1320

SALES AGENCIES

NASA Center for AeroSpace Information (CASI)
Parkway Center
7121 Standard Drive
Hanover, MD 21076-1320
UNITED STATES

The British Library Document Supply Centre
Boston Spa, Wetherby
West Yorkshire LS23 7BQ
UNITED KINGDOM

Canada Institute for Scientific and Technical Information (CISTI)
National Research Council
Acquisitions
Montreal Road, Building M-55
Ottawa K1A 0S2, CANADA

Requests for RTO or AGARD documents should include the word 'RTO' or 'AGARD', as appropriate, followed by the serial number (for example AGARD-AG-315). Collateral information such as title and publication date is desirable. Full bibliographical references and abstracts of RTO and AGARD publications are given in the following journals:

Scientific and Technical Aerospace Reports (STAR)
STAR is available on-line at the following uniform resource locator:
http://www.sti.nasa.gov/Pubs/star/Star.html
STAR is published by CASI for the NASA Scientific and Technical Information (STI) Program
STI Program Office, MS 157A
NASA Langley Research Center
Hampton, Virginia 23681-0001
UNITED STATES

Government Reports Announcements & Index (GRA&I)
published by the National Technical Information Service
Springfield
Virginia 2216
UNITED STATES
(also available online in the NTIS Bibliographic Database or on CD-ROM)

ISBNs 92-837-0059-7 / 978-92-837-0059-3