# Overview

Phenomena referred to generally as combustion instabilities are fundamentally related to the stability of motions in a combustion chamber. Their existence is normally inferred from observations of well defined oscillations of pressure or structural distortions. Instabilities of combustion processes themselves are rarely contributing factors, the chief exceptions being possible intrinsic instabilities of solid propellants, and the weakening of flame stabilization mechanisms near the lean operating limits of gaseous and liquid fueled systems. Broadly, then, the appearance of a combustion instability is due to a loss of stability of the composite dynamical system comprising the combustion processes and the chamber itself, containing the medium which supports waves associated with the unstable motions.

Combustion instabilities have been found in all types of systems. The reason is simple and fundamental: In a combustor, by design the combustion processes generate high power densities under conditions when the losses of energy are small. Only weak coupling between fluctuations of the combustion power and the flow of the medium is sufficient to produce undesirable fluctuations of pressure and kinetic energy in the flow. Mainly three characteristics of a system influence its dynamical behavior: the physical state in which reactants are introduced (solid, liquid, gas); the geometry of the system; and the specific mechanism causing the instabilities to occur. It is therefore possible to construct a general analytical framework sufficiently comprehensive to capture most, if not all, of the main features of instabilities in any combustion system. One purpose of this book is to describe an approach that has been applied successfully to a broad range of problems arising in laboratory devices and in full-scale systems.

The approach follows a well-travelled path which consists essentially in constructing reduced-order models of dynamical behavior by first applying a method of spatial averaging. Formally the dynamics of the continuous flow system having an infinite number of degrees of freedom is represented by the dynamics of a system of coupled nonlinear oscillators in one-to-one correspondence with the natural acoustic modes of the combustor in question. Long understood from experience in several fields, the idea leads to an analytical framework that is easily applied to laboratory test devices, sub-scale tests and to full-scale combustion systems of all sorts. Representation constructed of acoustic modes is not so restrictive as first appears, neither in respect to the perturbing processes, nor in respect to linearity. So long as the amplitudes are not 'too' great, quite general motions can be synthesized. For realistic applications, the greatest difficulties arise in modeling the dominant physical and chemical processes; and in determining certain material and dynamical properties.

I make no attempt in this work to give a survey of possible methods of analysis; nor do I cover much more than the single method, with major applications. In particular, I do not discuss the many analyses based on solutions to partial differential equations. Considerable experience has demonstrated that relying on the averaged equations is an effective strategy both for understanding physical behavior and for obtaining useful results for practical problems. But I certainly do not claim any sort of universality.

This book ranges over a broad spectrum of topics from the physical foundations, experimental results and mathematical methods, to summary examinations of some experiences with combustion instabilities in

operational systems. The text is organized roughly into three main parts. Chapters 1 and 2 cover practical examples of combustion instabilities; their interpretation in elementary terms; and lengthy discussions, in Chapter 2, of the chief mechanisms for instabilities in laboratory and full-scale systems. A substantial part of Chapter 2 is devoted to unsteady combustion in solid propellant rockets. Some may consider that the emphasis is misplaced particularly because there is at the present time much greater concern with instabilities and oscillations in gas turbines, covered cursorily in Chapter 9.

There are good reasons for the space devoted to basic problems arising in solid rockets. Practical considerations forced by combustion instabilities have been present since the late 1930s; with changes of design and propellant systems they continue and will likely always be present. Perhaps the most important reason for such intense theoretical considerations is the fundamental property that a solid rocket, or a test sample of solid propellant, can be fired only once. There is accordingly enormous motivation, always present in planning test programs based on solid propellants, for maximizing the information gained from a single firing. That explains the concern with transient behavior, the periods of growth and decay of oscillations. The rates of change exhibited in the envelopes of oscillatory motion are determined by the averaged influences of all contributing physical processes. To understand those rates and their combined consequence, requires attention to the basic behavior of the system, a central motivation throughout this book.

An interesting peripheral issue in the history of solid rockets is the matter of national security and classification. In the very late 1950s and early 1960s, success was achieved in releasing some material for late publication and in relaxing restrictions on availability of current work. The story of that process is part of the main subject here because official rules may have direct effect on the style certainly, but also on the content and quality of research. Section 2.2 serves partly to make the point in respect to combustion instabilities in solid rockets.

Chapters 3, 4 and 5 form the second part of the book, covering largely theoretical and analytical matters. The conservation equations for a two-phase flow are developed in Annexes A and B. Their approximation by a single fluid model is the basis in Chapter 3 for extracting systems of equations for the averaged and time-dependent motions. The latter are then simplified by expansion in two small parameters, characteristic Mach numbers of the mean and fluctuating flows. Five classes of problems are identified according to the orders of terms retained in the expansion; the classes include classical acoustics, linear stability, and three types of nonlinear problems.

The method of approximate analysis used in this book is developed in Chapter 4. In outline it follows well-known strategies for analyzing dynamical systems: A modal expansion; spatial averaging with suitable weighting functions, chosen here to be the unperturbed acoustic modes; and, as an optional possible tactic for further simplifying the equations, application of time-averaging. The spatially averaged equations, a system of ordinary nonlinear second order equations, are solved by a perturbation-iteration procedure that produces systems of equations for the amplitudes of the modes, having forms systematically defined according to the two-parameter expansion procedures. The procedure is well-founded and has long been used in other fields.

An important property, long known but explicitly emphasized here, is that the results are **not** restricted to irrotational velocity fields. That true property is contrary to a criticism often incorrectly directed to this method, most recently in three papers given in August 2004. If the analysis is applied correctly (albeit additional modeling may be required), it is entirely capable of treating problems in which vorticity is important, always to some approximation. Moreover, also contrary to criticisms attempted several times in the literature, the approximate solution satisfies the correct perturbed boundary conditions, not those set on the basis functions used in the modal expansion. Annex F is included to address this point.

Owing to confusion and mis-interpretation of the points just made, I should be more specific. The following remarks are expanded in Chapter 4. For linear harmonic motions, the first order approximation in

$\bar{\mathbf{M}}_r$, the Mach number of the mean flow, is equation (4.82), which is (4.20) written in different symbols:

$$\hat{p}^{(1)}(\mathbf{r}) = \psi_N(\mathbf{r}) + \sum_{n=0}^{\infty}{}' \frac{\psi_n(\mathbf{r})}{E_n^2\,(k^2 - k_n^2)} \left\{ \iiint_V \psi_n(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_n(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.82}$$

The prime on the summation sign signifies that the $N^{th}$ term is missing. This expression represents the actual pressure field perturbed from the $N^{th}$ classical acoustic mode $\psi_N(\mathbf{r})$ by the processes accounted for in the perturbation functions. The corresponding formula for the perturbed Mach number field is (4.85), and the wavenumber of the actual motion is given by (4.19) written for the $N^{th}$ mode:

$$i\bar{\rho}\bar{a}k\mathbf{M}'^{(1)} = -\nabla\hat{p}^{(1)} - \varepsilon\mu\{[\hat{\mathbf{M}}]\}_1 + \varepsilon\hat{\boldsymbol{\mathfrak{F}}}_{10} + \varepsilon\mu\hat{\boldsymbol{\mathfrak{F}}}_{11} \tag{4.85}$$

$$k^2 = k_N^2 + \frac{1}{E_N^2} \left\{ \iiint_V \psi_N(\mathbf{r}_0)\hat{h}(\mathbf{r}_0)dV_0 + \oiint_S \psi_N(\mathbf{r}_{0s})\hat{f}(\mathbf{r}_{0s})dS_0 \right\} \tag{4.19}$$

What has apparently been confusing is the well-known property of this kind of perturbation/iteration procedure that the first order properties of the *actual* $N^{th}$ mode can be calculated if the properties of the $N^{th}$ *unperturbed* mode are known. Thus in the work here, besides the perturbations $\hat{h}$ and $\hat{f}$, only the wavenumber and mode shape of the $N^{th}$ unperturbed classical acoustic mode are required to calculate the properties, including the wavenumber (4.19), of the actual *perturbed* $N^{th}$ mode. To this level of approximation one does not need to know explicitly, for example, the pressure and velocity fields of the actual modes treated. But nevertheless they are implicit. In particular, the classical unperturbed modes are for irrotational flow but the flows of actual perturbed modes are rotational, deducible, for example, from (4.82) and (4.85). The wavenumber (4.19) is therefore the wavenumber of a rotational wave.

Results for a given problem require the functions $\hat{h}(\mathbf{r})$ and $\hat{f}(\mathbf{r})$. The procedure developed in Chapters 3 and 4 is intended only to provide a framework within which particular forms of $h$ and $f$ are placed; the forms themselves must be worked out as a separate chore for a chosen problem.

A few words about the property that the method really is 'approximate' are needed. There are two small parameters measuring smallness in the method: A characteristic average Mach number $\bar{\mathbf{M}}_r$ measuring in the first instance the intensity of the mean flow; and a Mach number $\mathbf{M}_r'$ indicative of the amplitude of fluctuations, most clearly the amplitude of acoustic velocity waves. It is the nature of perturbation methods that accuracy—somehow defined by comparison with the 'true' results which are normally not known—is lost as the sizes of the small parameters are increased. The way in which accuracy deteriorates is simply not known, not is it investigated here, because the real value of the method often lies less with its accuracy than with the ease with which it may produce results indicative of trends produced by changes in a system.

In Chapter 5, classical acoustics is reviewed, chiefly to summarize those results most relevant to problems of combustion instabilities. Much of the material is well-known and available in many texts, but some special topics are included to give a fairly complete muster of the purely acoustical results most commonly useful in the field of combustion instabilities.

The third part, the remainder of the body of this book, includes subjects common to many engineering fields but covered here with emphasis on applications to combustion systems: linear stability (Chapter 6); nonlinear behavior (Chapter 7); passive control (Chapter 8); and feedback control (Chapter 9). Examples of results and, when possible, comparison of predictions with observations, are scattered throughout those four chapters, to convey a sense of the current state of the field; and to suggest some of the areas where effort is required to achieve continued progress. Chapters 6 and 7 contain much material which is basic to the subject and is covered in some detail. In contrast, Chapters 8 and 9 are mainly descriptive. Chapter 8 is an

incomplete survey of the principal methods of passively controlling combustion instabilities, the important measures taken in practice; the last section of the chapter covers a simple example to illustrate the matter for an idealized thrust augmentor. Chapter 9 is an introduction to feedback control of combustion systems. I had intended to cover much more, but owing to constraints on time of preparation, the discussion has been cut short.

Although the idea of trying to manage combustion instabilities by some means of active control was first suggested by W. Bollay in 1951 and investigated in detail by Professor H.S. Tsien in 1952, results in the laboratory and in full-scale systems were not achieved until the past fifteen years or so. Despite the early enthusiasm, and subsequent achievement of a few impressive results, feedback control is still far from routine applications. Chapter 9 is included to provide a sketchy idea of how far the subject has progressed; and some indication of how much remains to be done. I believe that significant basic progress towards routine practical applications in this area awaits clearer basic understanding of the systems to be controlled. The subject merits continued attention, particularly at the fundamental level. There are many opportunities for extended research, not merely directed efforts intended to produce short-term 'pay-offs'. Success will contribute much to practical applications as well as to basic understanding.

There are eight annexes. The first two deal with formation of the equations of motion. Annex A is a fairly detailed and complete derivation of the equations for multi-component reacting flows in which one component is liquid or solid and the remainder are gases. The equations are combined to give the set governing flows of an equivalent mass-averaged fluid. Annex B gives the basis for the one-dimensional approximation, extremely important for practical applications. Annexes C and D cover topics which are important for special problems, but which don't fit well in the body of the text. Annex C establishes the correct formula for viscous attenuation of a planar acoustic wave at a rigid surface by using the one-dimensional approximation. It's remarkable that the result is exact. Equally important, this is the basis for the 'pumping' process accompanying unsteady conversion of solid to gas at a burning surface, a topic treated in Chapter 6. Some general aspects of vorticity and entropy fluctuations are covered in Annex D. It is often useful to model combustion zones as flame sheets; special considerations may arise when that approximation is used with spatial averaging. Examples are treated in Annex E. Annex F is included to help clarify some of the reasons that the approximate method of solution (Chapter 4) works well. The discussion is intended mainly to suggest how it is that the expansion for the acoustic field to first and higher order satisfies the correct perturbed boundary conditions even though the basis functions do not. Annex G is an extended discussion of the Nyquist Criterion, with examples, intended to provide part of the background required for understanding some of the works on feedback control covered or cited in Chapter 9. The last annex, H, is included to call attention to methods related to the method of averaging and used in other fields as approximations to nonlinear behavior.

It perhaps comes as a surprise to many that computational methods, especially computational fluid dynamics (CFD), are given almost no space in this book except for references. There are several reasons, mainly the fact that to the present time CFD has not been part of the mainstream of work on combustion instabilities. That circumstance is probably due partly to the backgrounds of the people working in the field, but largely because until rather recently, CFD would have offered little help for solving practical problems in which combustion instabilities are central. However, as in many fields of engineering, the situation is changing rapidly.

I certainly don't intend to imply that numerical computations of reacting unsteady flows have not been attempted. There are many examples. For instance, some of the earliest efforts are cited in NASA SP-194, "Liquid Rocket Instability" covering work carried out in the 1960s in support of the Apollo Program. In the early 1970s computation of transient unstable oscillations in solid rockets were limited to a few periods by the machines available. Much progress has been made since that time. For both research and practical purposes, constraints remain still but are rapidly becoming less severe. Nevertheless, even when CFD reaches the level

of providing perfect simulations of real problems, we will still confront the problem of understanding. I believe that analysis—especially approximate methods—will always fill that basic need.

The main reason that CFD has otherwise been relatively helpless in this subject is that problems of combustion instabilities involve physical and chemical matters that are still not well understood. Moreover, they exist in practical circumstances which are not readily approximated by models suitable to formulation within CFD. Hence, the methods discussed and developed in this book will likely be useful for a long time to come, in both research and practice.

In the past decade especially, increasingly detailed simulations have appeared, providing ever more faithful approximations to unsteady flows in combustion chambers. Those developments are particularly important in the field of solid propellant rockets for which it is necessary to deal with quite elaborate internal geometries. The significant task remains to incorporate in analysis using CFD both the realistic nonlinear behavior and valid models of unsteady (and eventually nonlinear) combustion dynamics. It seems to me that eventually the most effective ways of formulating predictions and theoretical interpretations of combustion instabilities in practice will rest on combining methods of the sort discussed in this book with computational fluid dynamics, the whole confirmed by experimental results.

Probably at this time the most promising formulation of CFD for unsteady flows in combustors is some form of large eddy simulation (LES). Some quite good results have been obtained for realistic geometries, although the methods are far from complete and are certainly not yet available for routine computations or practical design work. The flow fields computed always have characteristics which apparently can be interpreted as turbulent motions similar to those found in real combustors. How faithfully the actual fields are reproduced remains an open question.

As a practical matter, the methods used in this book fall somewhere between CFD and the semi-empirical methods commonly used in design work, at least in the initial stages. There is often—especially, but not only, in industry—a tendency to avoid practicing analysis, particularly in carefully constructed approximate forms, in favor of more appealing (dazzling?) methods requiring extensive computing resources, which may produce appealing multi-colored pictures. Extraordinarily important for many reasons, CFD in its various forms must be developed as far as possible. But suppose CFD could produce perfect results entirely equivalent to perfect experimental results. For some, that would evidently be a state of Nirvana. But others wonder whether our understanding of physical behavior would be correspondingly expanded. The sorts of results discussed in this book, and the procedures followed to obtain them, help serve the implied purpose.

On the other hand, I have also avoided in this book any calculations involving turbulence. That practice raises a serious question: Are the results computed with no effects of turbulence valid for practical flows which inevitably are highly turbulent? Certain experimental results and measurements carried out for full-scale devices have long suggested that narrowly and tentatively the answer is 'yes'. On the basis of fundamental considerations by Chu and Kovasznay (1957), discussed briefly in Annex D and Sections 3.1 and 7.9, a somewhat more definitive positive answer is available. For most of the purposes here, it seems that the influences of turbulence can be safely overlooked. (But see Section 7.9.) One must especially be aware of the assumption when experimental and theoretical results for variations in time, or the corresponding spectra, are compared, for it is surely true that eventually the effects of turbulence must be accounted for.

If the preceding is accepted as a reasonable interpretation of the broad development of the subject, a corollary conclusion is that convenient approximations to actual behavior are essential. The ability to capture the essence of a phenomenon in a short statement, or perhaps a formula, is evidence of understanding. Naturally one must always possess as well some understanding of the limitations of approximations if they are to be truly useful. An attractive feature of the methods discussed here is the ease with which one may deduce such "rules of thumb." Examples are scattered throughout the text.

In this book I try to give a fair idea of the historical aspects of combustion instabilities from roughly 1950 to the present. I hope that the foundations of the subject are sufficiently well covered that the essentials of the phenomenon in any type of chemical propulsion system may be understood. Although I have tried to include examples of instabilities in all types of propulsion systems now used, I make no claims of completeness. I realize that for want of space, or to avoid excessive reproduction of readily accessible material, I have excluded many good examples. To the best of my knowledge, I have tried to give proper credit to the principal contributors associated with the various fields. I have included more extensive attributions, as well as wider coverage, of certain parts of the material, in three versions of a short course devoted to the subject (Culick 2000a, 2001, 2002a).

Finally, I must vent a few comments on the problem of financial support by government agencies and others. Not surprisingly, as a subject which so often deals with problems, not desirable attributes, of practical systems, combustion instabilities has traditionally become a 'hot topic' only when there are serious problems in costly systems. Too often, support has been given for short-term quick fixes, and then has been withdrawn when the problems are managed. It is apparently not fertile ground for research program managers who seek to support career-building 'break-throughs'. Steady progress based partly on advances in contributing fields such as instrumentation, measurement techniques and computational resources generally, unfortunately, fails to offer obvious prospects of stardom. My own experience is that the subjects discussed in this book capture much of the basic ideas and materials common to broad areas of applied science and engineering. Consequently, even during periods when there are no particularly pressing practical problems, there is a need and ample justification for a well-grounded research program having rather general applications, and meriting continuous support over many years. Only in this way will there be a (small) community educated as required and having experience in the fields of practical applications covered here.

This book has depended on many works and direct help of others, beginning with my Sc.D. dissertation and my supervisor Professor Morton Finston (1919–1986) who gave me complete freedom in my choice of topic and in details of development, both of the subject and of myself. To him I remain forever grateful. Professor F.E. Marble sponsored my move out of student life at MIT to Caltech and from the beginning to the present has been generous with his advice, critiques and cogent opinions. During the time our careers touched, I benefitted much from friendship with Professor E.E. Zukoski (1927–1995). We collaborated most closely in projects involving the Caltech dump cumbustor; the best example is covered in Section 7.8. Mr. E. W. Price, then at the Naval Ordinance Test Station (1965), first gave me a long-term consulting arrangement, but more importantly, a seemingly endless supply of ideas, problems, and knowledgeable discussions. His personal contributions to the field of solid rocketry and his influence on its development are his unique legacy. Ed's careful reading of Chapter 2, especially Sections 2.1 and 2.2, followed by his lengthy and carefully written comments (initially a mixed review!) happily caused me to make significant changes greatly improving the text. Once again I am sincerely grateful to him for his friendship and help.

Professor M.W. Beckstead was first a co-author with me when he was working at NOTS, and later, when he had joined Hercules, Inc., gave me intellectual and financial support for my first work on nonlinear combustion instabilities, leading eventually to my 1974 paper. He has been a collaborator and co-author for many years. Dr. R. L. Derr was, and still is, a close supporter and friend whose opinion I value second to none. We share more than three decades of good memories ranging from late-night chats to a memorable low-level pass over Death Valley. I first met Professor Gary Flandro when he was a doctoral student at Caltech; we have remained mutual admirers and friends, sharing many interests including aeronautical history and model aircraft. We continue our professional collaboration and friendly competition. Mr. Jay Levine and I first became acquainted in the early 70s when I helped him a little with the basis of some of the first numerical calculations of unsteady motions in solid propellant rockets. Our relationship has always been and remains close and respectful. Mr. Norman Cohen was long a professional friend, but in the past few years, through the Caltech MURI program, we have worked closely and profitably; I have benefitted enormously. Professor

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I owe thanks to a huge number of professional colleagues and friends who are nameless here, yet who appear as authors in the list of references. I have chosen not to list you specifically so as not to risk causing prickly feelings by omissions. You surely know who you are—living in many countries! Be assured that I feel very deeply the gratitude I owe for what I have learned from you and for the many ways in which you have helped me. In particular, I have benefitted greatly from more than ten years' service on the AGARD Propulsion and Energetics Panel; six years' membership of the Sverdrup Advisory Council for AEDC; and from my current participation in the activities of the Technical Advisory Committee for Pratt and Whitney, Inc.

Before this work was published, I circulated a 'nearly final' draft to a few of the people listed above. I am grateful for their corrections, helpful comments and encouragement. Especially, Mr. Paul Kuentzmann of ONERA obviously spent much time reading the draft; I am grateful to him for his many comments and thoughtful suggestions. The points of view expressed in the text, and any errors of commission or omission, are of course my responsibility.

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