CHAPTER 1

Combustion Instabilities in Propulsion Systems

Chemical propulsion systems depend fundamentally on the conversion of energy stored in molecular bonds to mechanical energy of a vehicle in motion. The first stage of the process, combustion of oxidizer and fuel, takes place in a vessel open only to admit reactants and to exhaust the hot products. Higher performance is achieved by increasing the rate of energy release per unit volume. For example, the power densities in the F-1 engine for the Apollo vehicle (1960s), and the Space Shuttle main engine (1970s) are respectively 22 gigawatts/m^3, and 35 gigawatts/m^3. The power densities in solid rockets are much less. For a cylindrical bore, the values are approximately $0.25(r/D)$ gigawatts/m^3, where r is the linear burning rate, typically a few centimeters per second, and D is the diameter. Thus the power densities rarely exceed one gigawatt/m^3. An afterburner on a high-performance fighter may burn fuel at the rate of 75,000 pounds per hour, generating roughly 450 megawatts for a short period in a volume of perhaps (1 m^3), giving power densities around 0.3–0.4 gigawatts/m^3.

These are indeed very large power densities. To appreciate *how* large, consider the fact that the average power consumption per person in a developed country is about 4 kilowatts (roughly the same as that for astronauts). In the United States, with approximately 195 million people in 1965, the total power consumption was about 1,000 gigawatts. Hence, for a few minutes, the five F-1 engines in the first stage of the Apollo produced power equivalent to nearly 1% of the entire power consumption of the U.S. at that time — in a *very* small volume. We cannot be surprised that such enormous power densities should be accompanied by relatively small fluctuations whose amplitudes may be merely annoying or possibly unacceptable in the worst cases.

We are concerned in this book with the dynamics of combustion systems quite generally. The motivation for addressing the subject arises from particular problems of combustion instabilities observed in all types of propulsion systems. By "combustion instability" we mean generally an oscillation of the pressure in a combustion chamber, having a fairly well-defined frequency which may be as low as 10–20 Hz or as high as several tens of kilohertz. Typically the instabilities are observed as pressure oscillations growing spontaneously out of the noise during a firing. As a practical matter, combustion instabilities are more likely encountered during development of new combustion systems intended to possess considerable increases of performance in some sense. The present state of theory and experiment has not provided a sufficiently strong foundation to provide a complete basis for prediction. Hence there are only a few guidelines available to help designers avoid combustion instabilities.

Under such conditions, it is extremely important to pay attention to the experience gained in the laboratory as well as in full-scale tests of devices. Moreover, because of the many properties of the behavior common to the various systems, much is to be gained from understanding the characteristics of systems other than the one that may be of immediate concern. It is therefore proper to begin with a survey of some typical examples drawn from many years' experience. Theory is an indispensable aid to making sense of observational results. Conversely, discussion of various experimental observations is a natural place to introduce many of the basic ideas contained in the theory.

From the beginning of this subject, the central practical question has been: What must be done to eliminate combustion instabilities? Traditionally, the approach taken has been based on passive measures, largely *ad hoc* design changes or notably for solid propellant rockets, favorable changes of propellant composition. During the past few years, considerable effort has been expended on the problem of applying active feedback control to combustion systems. It's an attractive proposition to control or eliminate instabilities with feedback control, particularly because one implication, often made explicit, is that the use of feedback will somehow allow one to get around the difficult problems of understanding the details of the system's behavior. Many laboratory, and several full-scale demonstrations, apparently support that point of view. However, for at least two reasons, serious application of feedback control must be based on understanding the dynamics of the system to be controlled:

(i) all experience in the field of feedback control generally has demonstrated that the better the controlled plant is understood, the more effective is the control;
(ii) without understanding, development of a control system for a full-scale device is an *ad hoc* matter, likely to involve expensive development with neither guarantee of success nor assurance that the best possible system has been designed.

Therefore we begin this book with a survey of combustion instabilities observed in various systems. The theoretical framework is constructed to accommodate these observations, but later emerges also as a perfect vehicle for investigating the use of active feedback control (Chapter 9).

1.1. Introduction

For the kinds of propulsion systems normally used, combustion chambers are intended to operate under conditions that are steady or change relatively slowly. The central questions addressed here concern the stability and behavior subsequent to instability of steady states in combustors. If a state is unstable to small disturbances, then an oscillatory motion usually ensues. Such combustion instabilities commonly exhibit well-defined frequencies ranging from 15 Hz or less to many kilohertz. Even at the highest amplitudes observed in practice, the instabilities consume only a small fraction of the available chemical energy. Thus, except in extremely severe instances, the oscillations do not normally affect the mean thrust or steady power produced by the systems. Serious problems may nevertheless arise due to structural vibrations generated by oscillatory pressures within the chamber or by fluctuations of the thrust. In extreme cases, internal surface heat transfer rates may be amplified ten-fold or more, causing excessive erosion of the chamber walls.

An observer usually perceives an unstable motion in a combustion chamber as "self-excited," a consequence of the internal coupling between combustion processes and unsteady motion.[1] Except in cases of large disturbances (e.g. due to passage of a finite mass of solid material through the nozzle), the amplitude of the motion normally seems to grow out of the noise without the intrusion of an external influence. Two fundamental reasons explain the prevalence of instabilities in combustion systems:

i) an exceedingly small part of the available energy is sufficient to produce unacceptably large unsteady motions;
ii) the processes tending to attenuate unsteady motions are weak, chiefly because combustion chambers are nearly closed.

Those two characteristics are common to all combustion chambers and imply that the possibility of instabilities occurring during development of a new device must always be recognized and anticipated.

[1] An alternative explantion, that observed combustion instabilities are (or may be) motions driven by the noise present in combustors is explored in Section 7.9; see also Figure 1.34.

Treating combustion instabilities is part of the price to be paid for high-performance chemical propulsion systems. It is a corollary of that condition that the problem will never be totally eliminated. Advances in research will strengthen the methods for solution in practical applications, and will provide guidelines to help in the design process.

The fact that only a small part of the total power produced in a combustor is involved in combustion instabilities suggests that their existence and severity may be sensitive to apparently minor changes in the system. That conclusion is confirmed by experience. Moreover, the complicated chemical and flow processes block construction of a complete theory from first principles. It is therefore essential that theoretical work always be closely allied with experimental results, and vice versa. No single analysis will encompass all possible instabilities in the various practical systems. There are nevertheless many features common to all combustion chambers. Indeed, it is one theme of this book that the characteristics shared by propulsion systems in many respects dominate the differences. While it is not possible to predict *accurately* the occurrence or details of instabilities, a framework does exist for understanding their general behavior, and for formulating statements summarizing their chief characteristics. For practical purposes, the theory often serves most successfully when used to analyze, understand, and predict *trends* of behavior, thereby also providing the basis for desirable changes in design. Experimental data are always required to produce quantitative results and their accuracy in turn is limited by uncertainties in the data.

Special problems may be caused by combustion instabilities interacting with the vehicle. Because the frequencies are usually well-defined in broad ranges, resonances with structural modes of the vehicle or with motions of components are common. Perhaps the best known form of this sort of oscillation is the POGO instability in liquid rockets. Strong couplings between chamber pressure oscillations, low-frequency structural vibrations, and the propellant feed system sustain oscillations. The amplitudes may grow to unacceptable limits unless measures are taken to introduce additional damping. A striking example occurred in the Apollo vehicle. The central engine of the cluster of five in the first stage was routinely shut off earlier than the others in order to prevent growth of POGO oscillations to amplitudes such that the astronauts would be unable to read instruments. Comments on the vibrations and the early shut off may be heard in communications recorded during the launch phase of several Apollo missions.

In the U.S., and possibly in other countries, notably Germany and Russia before and during World War II, combustion instabilities were probably first observed in liquid rocket engines. Subsequent to the war, considerable effort was expanded in Russia and in the U.S. to solve the problem, particularly in large engines. Probably the most expensive program was carried out during development of the F-1 engine for the Apollo vehicle in the years 1962–1966, reviewed in a useful report by Oefelein and Yang (1993).

Liquid-fueled air-breathing propulsion systems also commonly suffer combustion instabilities. Axial oscillations in ramjet engines are troublesome because their influence on the shock system in the inlet diffuser can reduce the inlet stability margin. Owing to their high power densities and light construction, thrust augmentors or afterburners are particularly susceptible to structural failures.

For any thrust augmentor or afterburner, conditions can be found under which steady operation is not possible. As a result, the operating envelope is restricted by the requirement that combustion instabilities cannot be tolerated. Due to structural constraints placed on the hardware, combustion instabilities in afterburners are particularly undesirable and are therefore expensive to treat.

In recent years combustion instabilities in the main combustor of gas turbines have become increasingly troublesome. The chief reason is ultimately due to requirements that emission of pollutants, notably oxides of nitrogen, be reduced. A useful strategy, particularly for applications to flight, is reduction of the average temperature at which combustion takes place. Generation of NO by the thermal or 'Zel'dovich' mechanism is then reduced. Lower combustion temperature may be achieved by operating under lean conditions, when

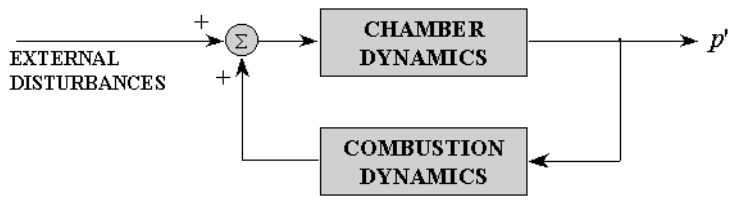


FIGURE 1.1. Schematic diagram of a combustion system as a feedback amplifier.

the flame stabilization processes tend to be unstable. Fluctuations of the flame cause fluctuations of energy release, which in turn may produce fluctuations of pressure, exciting acoustical motions in the chamber.

Finally, almost all solid rockets exhibit instabilities, at least during development, and occasionally motors are approved even with low levels of oscillations. Actual failure of a motor itself is rare in operations, but vibrations of the supporting structure and of the payload must always be considered. To accept the presence of weak instabilities in an operational system one must have sufficient understanding and confidence that the amplitudes will not unexpectedly grow to unacceptable levels. One purpose of this book is to provide the foundation for gaining the necessary understanding.

In the most general sense, a combustion instability may be regarded as an unsteady motion of a dynamical system capable of sustaining oscillations over a broad range of frequencies. The source of energy associated with the motions is ultimately related to the combustion processes, but the term 'combustion instability,' while descriptive, is misleading. In most instances, and always for the practical problems we discuss in this book, the combustion processes themselves are stable: Uncontrolled explosions and other intrinsic chemical instabilities are not an issue. Observations of the gas pressure or of accelerations of the enclosure establish the presence of an instability in a combustion chamber. Excitation and sustenance of oscillations occur because coupling exists between the combustion processes and the gasdynamical motions, both of which may be stable. What is unstable is the entire system comprising the propellants, the propellant supply system, the combustion products that form the medium supporting the unsteady motions, and the containing structure.

If the amplitude of the motions is small, the vibrations within the chamber are often related to classical acoustic behavior possible in the absence of combustion and mean flow. The geometry of the chamber is therefore a dominant influence. Corresponding to classical results, traveling and standing waves are found at frequencies approximated quite well by familiar formulas depending only on the speed of sound and the dimensions of the chamber. If we ignore any particular influences of geometry, we may describe the situation generally in the following way, a view valid for any combustion instability irrespective of the geometry or the type of reactants.

Combustion processes are sensitive to fluctuations of pressure, density, and temperature of the environment. A fluctuation of burning produces local changes in the properties of the flow. Those fluctuations propagate in the medium and join with the global unsteady field in the chamber. Under favorable conditions, the field develops to a state observable as a combustion instability. As illustrated schematically in Figure 1.1, we may view the process abstractly in analogy to a feedback amplifier in which addition of feedback to a stable oscillator can produce oscillations. Here the oscillator is the combustion chamber, or more precisely, the medium within the chamber that supports the unsteady wave motions. Feedback is associated with the influences of the unsteady motions on the combustion processes or on the supply system, which in turn generate fluctuations of the field variables. The dynamical response of the medium converts the local fluctuations to global behavior. In the language of control theory, the field in the chamber is the 'plant,' described by the general equations of motion.

The diagram in Figure 1.1 illustrates the main emphases of this book. Broadly, the subjects covered divide into two categories: those associated with the plant—the fluid mechanics and other physical processes comprising the *combustor dynamics;* and those connected primarily with the feedback path, chiefly combustion processes and their sensitivity to time-dependent changes in the environment, the *combustion dynamics.* The theory we will describe encompasses all types of combustion instabilities in a general framework having the organization suggested by the sketch. External forcing functions are accommodated as shown in the sketch, but the causes associated with the feedback path are far more significant in practice.

Figure 1.1 is motivated by more than a convenient analogy. For practical purposes in combustion systems, we generally wish to eliminate instabilities. Traditionally that has meant designing systems so that small disturbances are stable, or adding some form of energy dissipation to compensate the energy gained from the combustion processes, that is, *passive control.* However, in the past few years interest has grown in the possibility of *active control* of instabilities. If that idea is to be realized successfully, it will be necessary to combine modern control theory with the theory developed in this book. The development of much of control theory rests partly on the use of diagrams having the form of Figure 1.1. It is advantageous to think from the beginning in terms that encourage this merger of traditionally distinct disciplines.

We will return to the subject of control, both active and passive, in the last two chapters of this book. Any method of control is rendered more effective the more firmly it rests on understanding the problem to be solved. Understanding a problem of combustion instabilities always requires a combination of experiment and theory. For many reasons, including intrinsic complexities and inevitable uncertainties in basic information (e.g., material properties, chemical dynamics, turbulent behavior of the flow field, ...), it is impossible to predict from first principles the stability and nonlinear behavior of combustion systems. Hence the purpose of theory is to provide a framework for interpreting observations, both in the laboratory and full-scale devices; to suggest experiments to produce required ancillary data or to improve the empirical base for understanding; to formulate guidelines for designing full-scale systems; and globally to serve, like any good theory, as the vehicle for understanding the fundamental principles governing the physical behavior, thereby having predictive value as well.

All theoretical work in this field has been carried out in response to observational and experimental results. We therefore spend much of the remainder of this introductory chapter on a survey of the characteristics of combustion instabilities observed, and occasionally idealized, in the systems to be analyzed in later chapters.[2] The general point of view taken throughout the book will then be formulated in heuristic fashion, based on experimental results.

1.2. Historical Background

Some of the consequences and symptoms of combustion instabilities were first observed in the late 1930s and early 1940s, roughly at the same time for liquid and solid propellant rockets, and apparently somewhat earlier in the Soviet Union than in the U.S. With the later development of turbojet engines, high-frequency instabilities were found in thrust augmentors or afterburners in the late 1940s and early 1950s. Although the problem had been encountered in ramjet engines in the 1950s, it became a matter of greater concern in the late 1970s and 1980s. The introduction of compact dump combustors led to the appearance of longitudinal or axial oscillations that interfered with the inlet shock system, causing loss of pressure margin and 'unstart' in the most severe cases. Owing to availability, almost all of the data cited here as examples will be derived from liquid rockets, solid rockets and laboratory devices. Figure 1.2 is a qualitative representation of the chronology of combustion instabilities. Due to the accessibility of documentation and the experiences of the author, particular cases cited are mainly those reported in the U.S. We will be mainly concerned in this book

[2] A few references to papers and books are given in this chapter as guides to the literature. Later chapters contain more complete citations as the topics are treated in greater detail.

with oscillations in solid rockets; liquid and gas rockets; thrust augmentors or afterburners; ramjets; and, to a lesser extent, gas turbines.

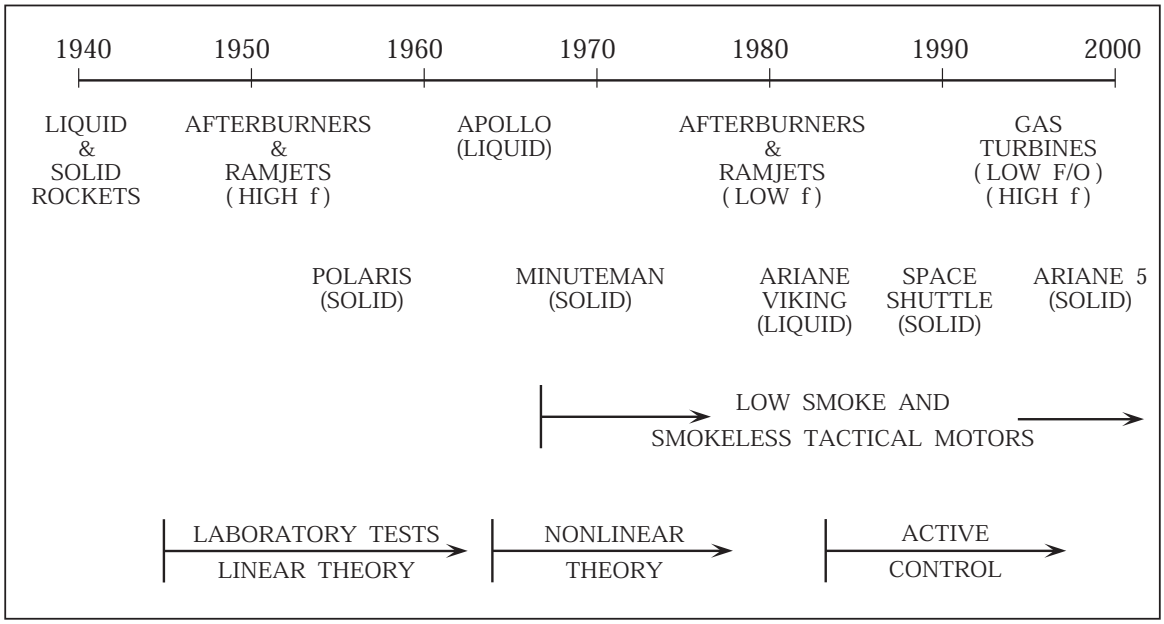


FIGURE 1.2. A chronology of combustion instabilities.

Several reviews of early experiences with combustion instabilities have been prepared for liquid rockets (Ross and Datner 1954) and for solid rockets (Wimpress 1950; Price 1968; Price and Flandro 1992). The details are not important here, but the lessons learned certainly are. Often forgotten is the most important requirement of good high-frequency instrumentation to identify and understand combustion instabilities in full-scale as well as in laboratory systems. Until the early 1940s, transducers and instrumentation for measuring pressure had inadequate dynamic response to give accurate results for unsteady motions. Ross and Datner note that "Prior to 1943, the resolution of Bourdon gauges, photographed at 64 and 128 fps, constituted the principal instrumentation." Recording oscillographs were introduced sometime in 1943, but not until the late 1940s were transducers available with sufficient bandwidth to identify instabilities at higher frequencies (hundreds of hertz and higher).

The situation was even more difficult with solid rockets because of the practical difficulties of installing and cooling pressure transducers. Probably the experience with cooling chamber and nozzle walls helps explain why quantitative results were obtained for instabilities in liquid rockets earlier than for solid rockets (E. W. Price, private communication). Prior to the appearance of high-frequency instrumentation, the existence of oscillations was inferred from such averaged symptoms as excessive erosion of inert surfaces or propellant grains due to increased heat transfer rates; erratic burning appearing as unexpected shifts in the mean pressure; structural vibrations; visible fluctuations in the exhaust plume; and, on some occasions, audible changes in the noise produced during a firing.

Experimental work progressed for several years before various unexplained anomalies in test firings were unambiguously associated with oscillations. By the late 1940s, there was apparently general agreement among researchers in the U.S. and Europe that combustion instabilities were commonly present in rocket motors and that they were somehow related to waves in the gaseous combustion products. In addition to measurements with accelerometers, strain gauges, and pressure transducers, methods for flow visualization soon demonstrated their value, mainly for studies of liquid propellant rockets (Altseimer 1950; Berman and

Logan 1952; and Berman and Scharres 1953). Characteristics of the instabilities as acoustic vibrations, or weak shock waves, were revealed.

It is much more difficult to observe the flow field in a solid rocket motor and during the early years of development, the only results comparable to those for liquid rockets were obtained when excessive chamber pressures caused structural failures. Partially burned grains often showed evidence of increased local burning rates, suggesting (possibly) some sort of influence of the gas flow. The same events also produced indications of unusual heating of the unburned solid propellant, attributed to dissipation of mechanical vibrational energy (Price and Flandro, 1992). Subsequently that interpretation was confirmed by direct measurements (Shuey, H.C. 1970-1975, related to the author by Mr. E.W. Price).

High-frequency or 'screech' oscillations were also first encountered in afterburners in the late 1940s; as a result of the experience with rockets and the availability of suitable instrumentation, the vibrations were quickly identified as combustion instabilities. The staff of the Lewis Laboratory (1954) compiled most of the existing data and performed tests to provide a basis for guidelines for design.

Thus by the early 1950s many of the basic characteristics of combustion instabilities had been discovered in both liquid-fueled and solid-fueled systems. Many of the connections with acoustical properties of the systems, including possible generation of shock waves, were recognized qualitatively. Although the frequencies of oscillations found in tests could sometimes be estimated fairly closely with results of classical acoustics, no real theory having useful predictive value existed. During the 1950s and the 1960s the use of sub-scale and laboratory tests grew and became increasingly important as an aid to solving problems of combustion instabilities occurring in the development of new combustion systems of all types.

1.2.1. Liquid and Gas-Fueled Rockets. During the 1960s, the major efforts on combustion instabilities in liquid rockets were motivated by requirements of the Apollo vehicle. Harrje and Reardon (1972) edited a large collection of contributions summarizing the work during that period. Essentially nothing additional was required to treat instabilities in the Space Shuttle main engine, and in the U.S., new programs specifically dealing with liquid rockets did not appear again until the mid 1980s (Fang 1984, 1987; Fang and Jones 1984, 1987; Mitchell, Howell, and Fang 1987; Nguyen 1988; Philippart 1987; Philippart and Moser 1988; Jensen, Dodson, and Trueblood 1988; and Liang, Fisher and Chang 1986, 1987.) Subsequent to a flight failure of an Ariane vehicle due to combustion instability in a first-stage Viking motor, a comprehensive research program was begun in France in 1981 (Souchier *et al.* 1982; Schmitt and Lourme 1982; Habiballah *et al.* 1984, 1985, 1988, 1991; Lourme *et al.* 1983, 1984, 1985, 1986).

The problem of instabilities in liquid rockets received greatest attention in the development of the man-carrying vehicles for flight to the moon. Both the U.S. and the U.S.S.R. expended great effort on the problem. Because virtually all the necessary information is conveniently available, we examine the experience in the U.S., a canonical example. Oefelein and Yang (1992, 1993) have given a good summary of the program; their effort is the basis of the remarks here.

Instabilities in liquid rockets are simpler than in solid rockets, to some extent because the basic internal geometry of the system is simpler. Serious complications are introduced by the flow and mixing of reactants. Practically the only influence that the existence of instabilities had on the configuration of the F-1 was on the form of axial baffles attached to the face of the injector at the head end of the motor; and on the details of the injector which influenced the interactions between the jets of fuel and oxidizer. Once the general form of injector had been chosen—impinging jets of fuel and oxidizer—the number of variables was significantly reduced. Nevertheless, considerable freedom in the details of the injector remained. Consequently, development of the F-1 from October 1962 to September 1966 involved more than 3200 full-scale tests. Approximately 2000 of those tests were conducted as part of Project First, the program carried out to solve the problems arising with combustion instability in the F-1.

During Project First, the global geometry of the chamber and nozzle was not a matter of concern; we shall assume the geometry as given and essentially fixed. Three basic injectors were initially examined, all involving impinging jets. All showed spontaneous instabilities, that is, unacceptable oscillations without any external disturbances. Figure 1.3 shows a typical pressure trace taken at the beginning of the program. Because it was known that baffles were effective for reducing the sort of transverse oscillations observed, a configuration was chosen with thirteen compartments on the injector face, formed by barriers oriented normal to the face of the injector, extending about 7.6 cm. downstream from the injector face. Figure 1.4(b) is a picture of the injector, about one meter in diameter.

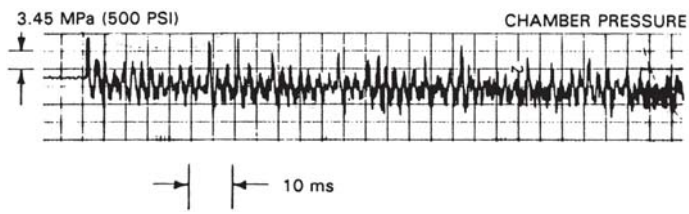
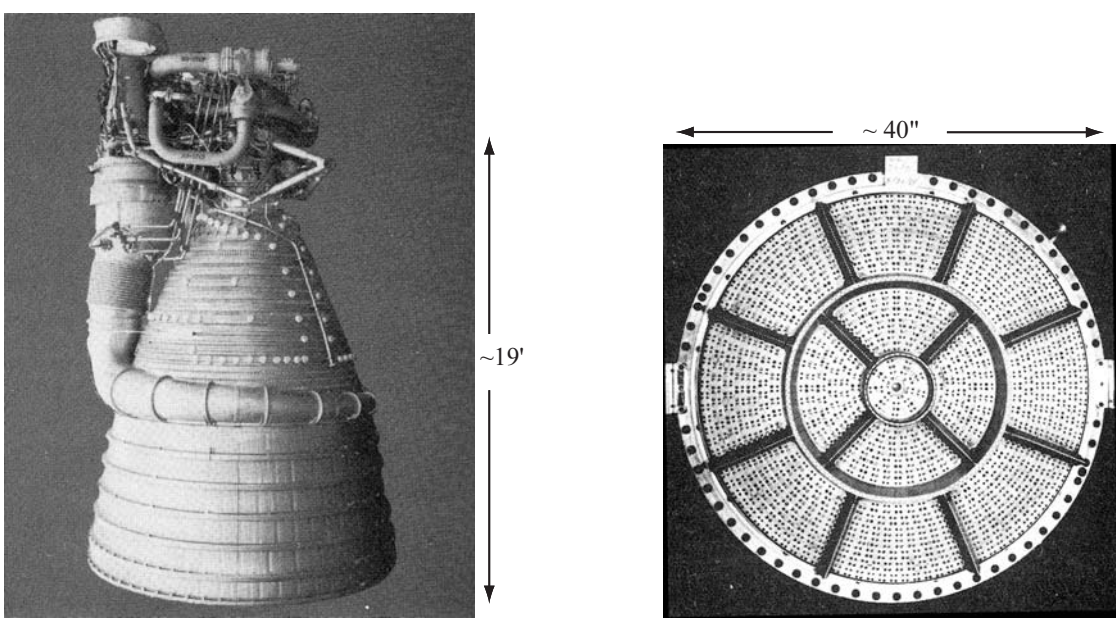


FIGURE 1.3. Pulses of the injected fuel stream in the F-1 engine (Oefelein and Yang 1993).

An informative collection of simplified pictures of the various sorts of injection elements was published by Rocketdyne, Inc. (Jaqua and Ferrenberg 1989), reproduced here in Figure 1.5. The configurations are of course sketched in simplified forms, but one important point is quite clear. If the formation of sheets and clouds of unlike drops depends on impingement of jets or liquid streams, then one expects the process to be sensitive to perturbations, particularly when there is a component of the disturbance normal to the plane of the sheet. Hence the effectiveness of baffles of the sort shown in Figure 1.4(b) is related to the shadowing of the injected streams.



(a) (b)

FIGURE 1.4. (a) The F-1 engine and (b) the face of the injector showing the fourteen baffles (Oefelein and Yang 1993).

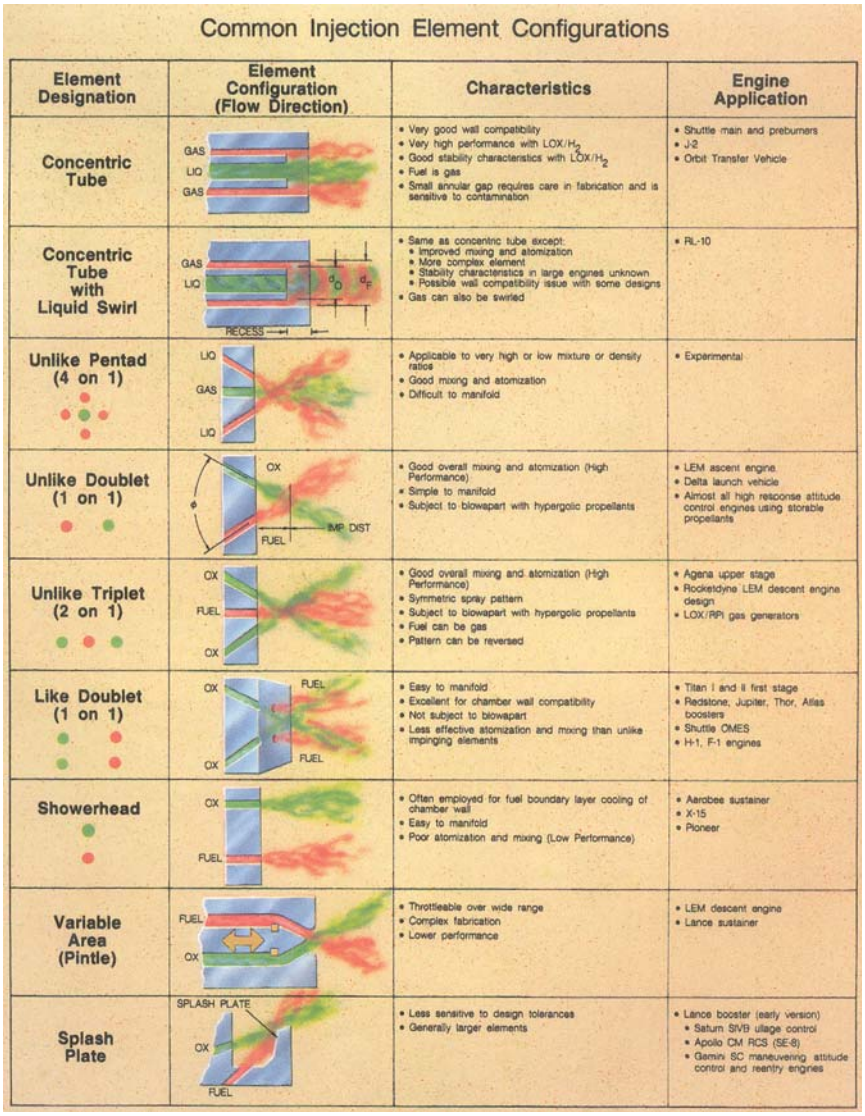


FIGURE 1.5. Injection elements commonly used in liquid rockets (Jaqua and Ferrenberg 1989).

The example of the F-1 and experience with reducing the combustion instability forms a basic canonical case. As shown in Figure 1.3, the instability had a frequency of about 440 Hz. and amplitudes greater than the mean chamber pressure in the chamber without baffles, but about 65% of the mean pressure in the chamber with baffles. Hence development proceeded with the baffles in place. Changes in the details of the injector design—such as hole sizes and the angles of impingement of the jets—eventually eliminated resurging. A method of pulsing or 'bombing' was then used to assess the stability of an injector, rated according to the damping time of a pulse. The procedure was entirely empirical since no theory existed (and still doesn't). During this stage, the phenomenon of 'resurging' was often observed as an instability, an example is shown in Figure 1.6. However, by November 1965, the engine exhibited acceptable damping times (c. 45 ms or less) following pulse amplitudes of acceptable magnitude. Figure 1.7 shows an example obtained with a flight qualified injector subjected to a 13.5 grain bomb.

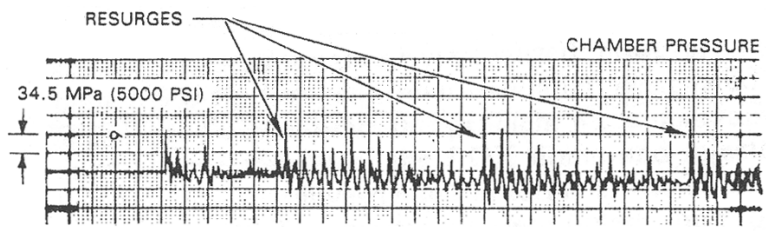


FIGURE 1.6. Resurging subsequent to a bomb-induced perturbation (Oefelein and Yang 1993).

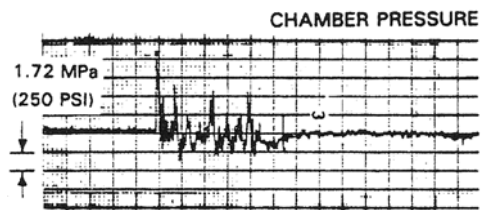


FIGURE 1.7. Decaying pressure induced initially by a pulse from a 13.5 grain bomb (Oefelein and Yang 1993).

There seems little doubt that the mechanisms for the instabilities were understood quite well in broad outline, if not in quantitative detail, for this kind of injector. Stretching downstream from the face of the injector there were three fairly well-defined regions. The first contains the spray fans generated by the injection jets; the next contains fuel drops breaking up and vaporizing; and the last, where only vapor exists, may be characterized by mixture ratio variations in space. Imposition of time-dependent properties, notably pressure and velocity, produces fluctuations of properties which in turn can generate fluctuations of the local burning rate. Thus all three regions can contribute to the tendency to instability.

The F-1 provides an example of an instability in a liquid oxygen (LOX) hydrocarbon (HC) system with injectors based on impinging jets. As the experience suggests, the configuration having jets impinging on one another seems to be quite sensitive to external perturbations. It appears for example, that the jets could be misaligned rather easily, leading possibly to an instability. Hence many designers have favored coaxial injectors, but it is not our purpose here to make a case for any particular kind of injector, which in any case may depend on the propellants in question. Perhaps the most common form of injector in general is based on coaxial flow; examples producing jets with and without swirl are shown in Figure 1.8. Summaries of recent research on coaxial injection elements in the context of combustion instabilities have been given by Hulka and Hutt (1995). They also give a good summary of U.S. experience from the late 1950s to the early 1990s.

An entirely different form of injector was introduced by TRW, Inc. for the lunar descent module of the Apollo vehicle; it was invented by G.W. Elverum at Caltech's Jet Propulsion Laboratory, and later developed at TRW (Elverum *et al.* 1967). The need for a throttleable engine was satisfied by a pintle design, sketched in Figure 1.9. Figure 1.10 is a photograph of a cutaway model of the engine. Thrust could be varied from 1000 to 10,000 pounds by moving the pintle.

The engine gained a deserved reputation for stability. Figure 1.11 shows the decay several pulses in a test of the engine at TRW. As a result of such testing, the engine has been widely viewed as 'absolutely' stable. However it is important to realize that one can state only that the engine is acceptably stable for the fuel and oxidizer used (nitrogen tetroxide and A-50, a 50/50 mixture of hydrazine and unsymmetrical dimethylhydrazine, called UDMH) and for a certain range of operating conditions. Limitations have been shown on more than one occasion, to the embarrassment of test program managers.

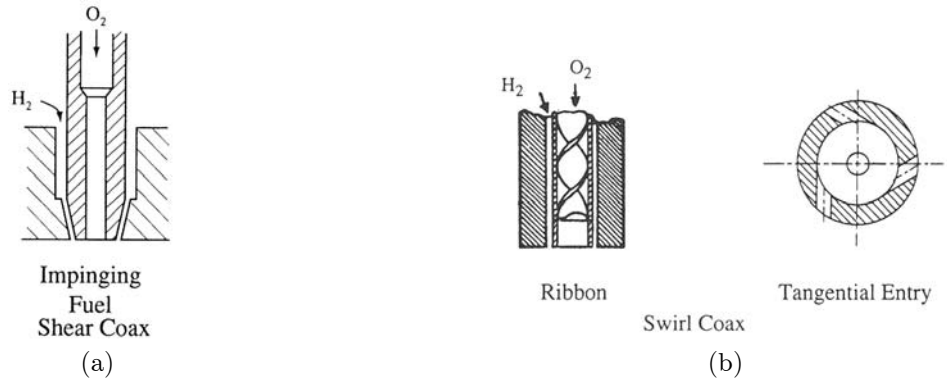


FIGURE 1.8. Simplified forms of coaxial injectors. (a) pure shear coaxial element; (b) coaxial element producing swirl (Hulka and Hutt 1995).

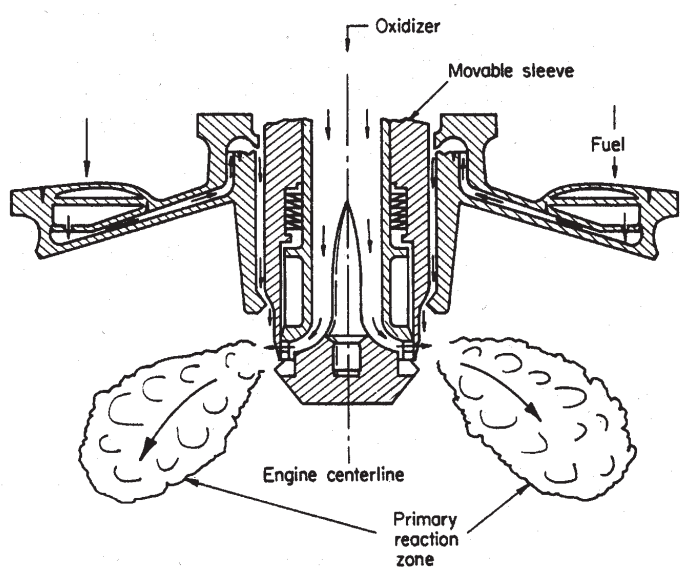


FIGURE 1.9. Pintle injector in the lunar excursion module (LEM) descent engine of the Apollo vehicle (Harrje and Reardon 1972).

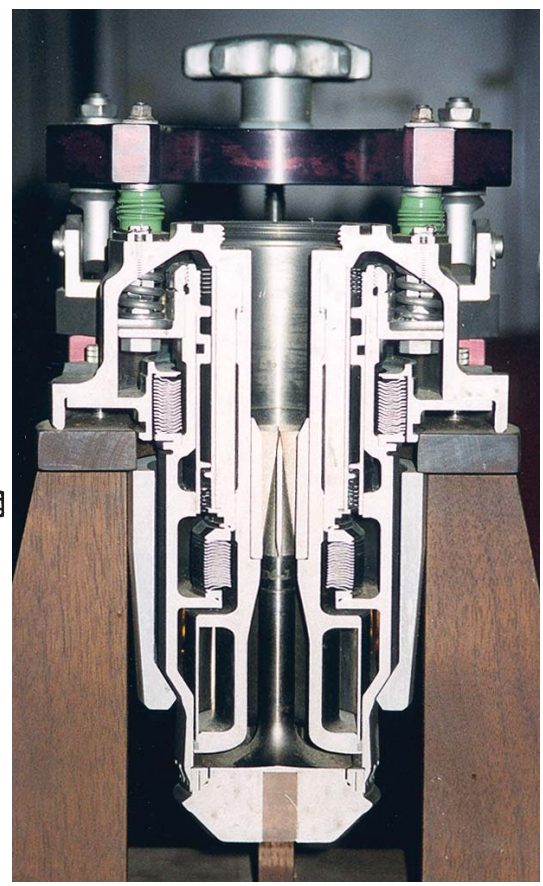


FIGURE 1.10. Cutaway display model of the mixing head of the TRW pintle engine (courtesy of Jack L. Cherne, TRW retired).

Instability problems also arose during development of the Space Shuttle main engine (SSME), which developed 500K pounds of thrust, but the difficulties were overcome. Three SSMEs are used on the Shuttle.

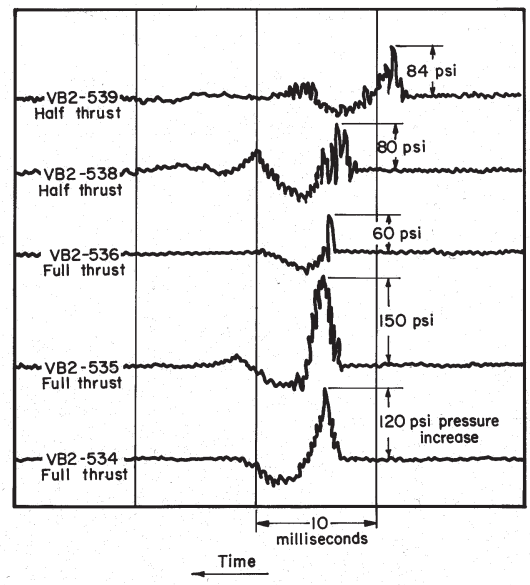


FIGURE 1.11. Decay of pulses injected in the TRW LEM engine (Harrje and Reardon 1972).

Figure 1.12 is a drawing of the engine showing the coaxial elements and the oxidizer elements serving also as baffles.

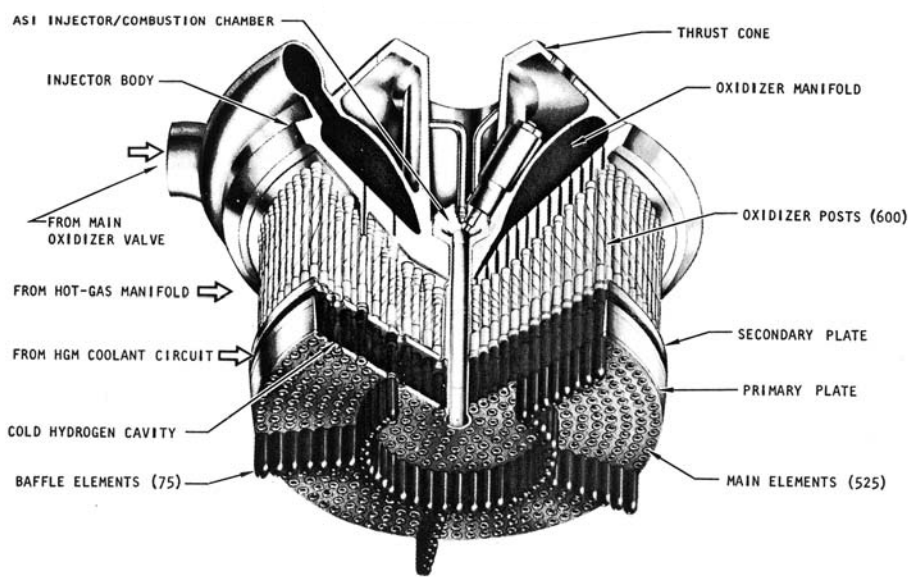
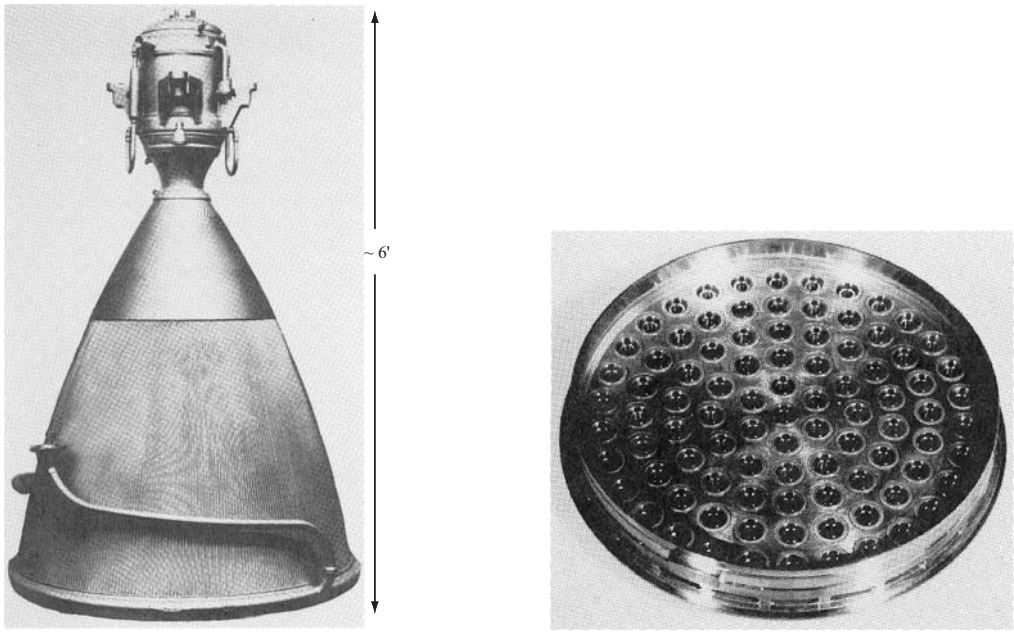


FIGURE 1.12. Cutaway drawing of the main injector assembly of the Space Shuttle main engine (Courtesy of Rocketdyne, Inc.).

Russian experience with combustion instabilities in liquid rockets was not well-known in the West until the 1990s although some publications were available, e.g. Natanzon (1984) which later appeared in English translation. The propulsion community in the U.S. generally did not learn of Russian research concerned



(a) (b)

FIGURE 1.13. The RD-0110 engine (a) general view of the basic engine; (b) the injector face (Rubinsky 1995).

with operational rockets until the conference held at Pennsylvania State University in 1993 ("Liquid Rocket Engine Combustion Instability", 1995). Subsequently an updated English translation of Natanzon's work appeared (Natanzon 1999). That work offers a good coverage of some fairly recent Russian work as well as a useful summary of many topics basic to the dynamics of liquid rockets. Russian experimental work on instabilities in liquid engines is covered in the book by Dranovsky (2006).

Rubinsky (1995) published what is likely the first thorough account of Russian experience with a problem of combustion instability in a liquid rocket engine, the RD-0110. The system used coaxial injectors (referred to as "bipropellant centrifugal atomizers") supplying liquid oxygen and a hydrocarbon fuel; the engine produced 67,000 pounds of thrust with a chamber pressure of 1000 psi. Chamber diameter is 7.1 inches, length 10.6 inches and throat diameter 3.3 inches. Figure 1.13 shows the engine and the face of the injector. The engine was used in the Soyuz vehicle with four motors powering the upper stage, Figure 1.14.

Extended development work eventually reduced the instability to a problem occurring once out of about 300 thrust chambers. It was a problem from ignition to 0.1 seconds or so. Installation of longitudinal ribs made of combustible felt solved the problem; Rubinsky (1995) describes the matter in detail. Figure 1.15 shows a chamber with ribs.

1.2.2. Combustion Instabilities in Thrust Augmentors. The situation in respect to instabilities in afterburners[3] seems to have changed little in fundamental respects in more than 20 years. Early work

[3]Increased thrust was obtained from early gas turbines by adding a second combustion chamber downstream of the turbine. The exhaust of the turbine is oxidizer rich, limitations being set on the mixture ratio for combustion (i.e., temperature) according to material properties. All of the flow entering the inlet then passed through both combustors; the second combustor was conventionally called an afterburner. With the development of bypass engines, the 'afterburner' received some air which had not passed through the main combustor. Hence the device became known as a 'thrust augmentor', referring to its purpose to augment the thrust, without implying the source of air.

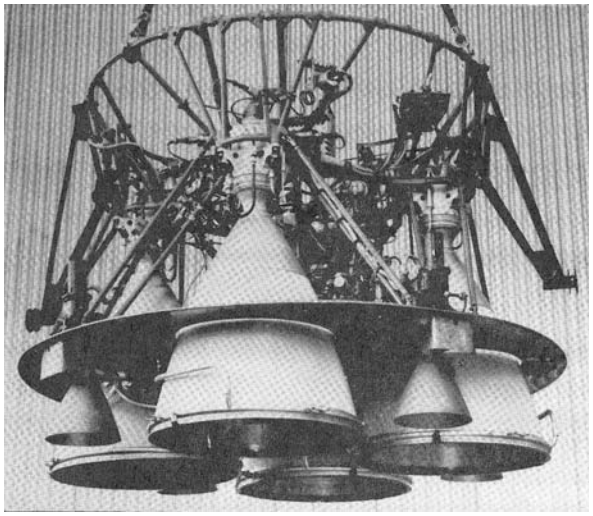


FIGURE 1.14. Four RD-0110 engines mounted in the upper stage of the Soyuz vehicle (Rubinsky 1995).

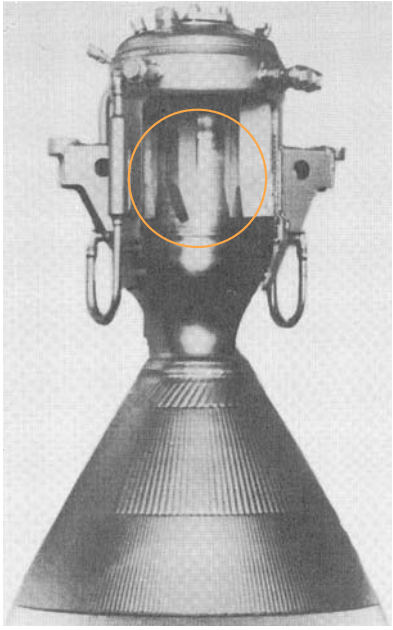


FIGURE 1.15. RD-0110 engine cutaway to show combustible ribs intended to attenuate a tangential mode (Rubinsky 1995).

showed that high-frequency oscillations ('screech') could be treated over fairly broad operating conditions by installing passive suppression devices—usually acoustic liners—and by adjusting the distribution and scheduling of the injected fuel. Problems increased as high-bypass engines were developed because the large annular flow passages allowed waves to propagate upstream to the compressor. As a result, instabilities occurred with longer wavelengths and hence lower frequencies (Bonnell *et al.* 1971; Kenworthy *et al.* 1974; Ernst 1976; Underwood *et al.* 1977; Russell *et al.* 1978). Figure 2.54 is a sketch showing an example of an augmented engine which exhibited low-frequency instabilities. Because considerable effort—and cost—has been spent to try to reduce or eliminate instabilities in augmentors by passive means, the subject will be examined in Chapter 8. See also Section 2.4.4.

Low frequencies are not easily attenuated, so modifications in the supply system and appropriate scheduling of the fuel injection are the main strategies for treating these modes. In any case, it appears that all afterburners are subject to operational constraints set by the need to avoid combustion instabilities. Both because of the operational constraints and because of the high costs incurred during development to give current operating envelopes, combustion instability in afterburners remains an attractive subject of research.

Broadly, then, the inevitable appearance of instabilities has led to a basic general strategy followed generally in the development of new afterburners. To the greatest extent possible, acoustic liners are installed in the lateral boundaries. These are effective for attenuating relatively high frequency oscillations historically called 'screech.' Such vibrations normally involve fluctuations in planes transverse to the axis of the burner, so the liners on the case of the burner work quite well.

Oscillations in the direction of flow are in general much more difficult to eliminate. In fact their presence seems often to set operating limits on the afterburner. Those limitations occur mainly in two regions of the basic operating envelope: at lower Mach numbers and high altitude; and at high Mach number and low

altitude. The limits are found from ground tests of the engine, a significant development cost for a new engine.

In general, current information for instabilities in afterburners is unavailable. The most recent work reported in the open literature deals with possible use of active control mainly to widen the operating envelope. An example of modest success at Rolls-Royce has been reported by Moran *et al.* (2001).

1.2.3. Combustion Instabilities in Ramjet Engines. Instabilities in liquid-fueled ramjet engines are similar in many respects to those in afterburners, an example of the general property that the character of the oscillations is determined largely by the types of propellants used and the geometry of the chamber. In both systems, the steady combustion processes are stabilized behind bluff flameholders. Hence with suitable interpretation, many results of research are applicable to both types of systems. In the late 1970s and 1980s, research programs on combustion instabilities in ramjet engines were initiated by several western countries (see, for example, Hall 1978; Culick 1980; Culick and Rogers 1980; Clarke and Humphrey 1986; Humphrey 1987; Sivasegaram and Whitelaw 1987; Zetterström and Sjöblom 1985; Biron *et al.* 1987; Culick and Schadow 1989). Figure 1.16 is a sketch of a stylized combustor representing the sort of configuration commonly used in liquid-fueled ramjets.

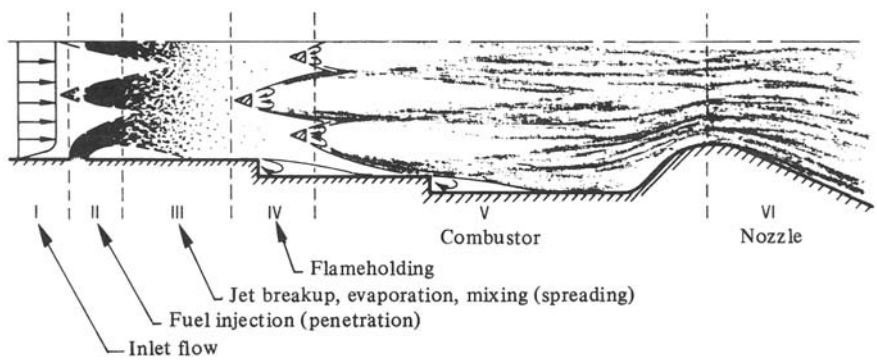


FIGURE 1.16. Stylized configuration of a liquid-fueled ramjet (United Technologies Corp. 1978).

Possibly the most interesting and fundamental result of work during that period was demonstration of the importance of coupling between acoustical motions and large coherent vortex structures shed by a rearward facing step or a flameholder, first emphasized by Byrne (1981, 1983). That phenomenon, with or without combustion processes, arises in many situations and will likely long continue to be the subject of research. Problems associated with generation of unsteady vorticity and vortex shedding arise in all types of combustors. Much effort has been expended in this area, a subject that will arise often in this text.

A typical example is that investigated thoroughly in the dump combustor at Caltech (Smith, 1985; Sterling, 1987; Zsak, 1993; Kendrick, 1995). Figure 1.17 is a sketch of the configuration, in which the flow is subsonic throughout, with premixed gaseous reactants introduced from a plenum chamber and exhausting to the atmosphere. Even for liquid-fueled ramjets, vaporization of the fuel often occurs so rapidly that combustion downstream of the dump plane occurs in a gaseous mixture. The general character of the stability diagram for this geometry has been found in other experimental programs as well: for a given flow rate, the most intense oscillations occur in the vicinity of stoichiometric proportions of the fuel oxidizer.

The waveform and spectrum for the limiting behavior of an unstable oscillation are shown in Figure 1.18. Evidently the spectrum consists of a small number of peaks imbedded in a background of 'noise' spread over the entire frequency range covered. In this respect the motion seems to be dominated by two

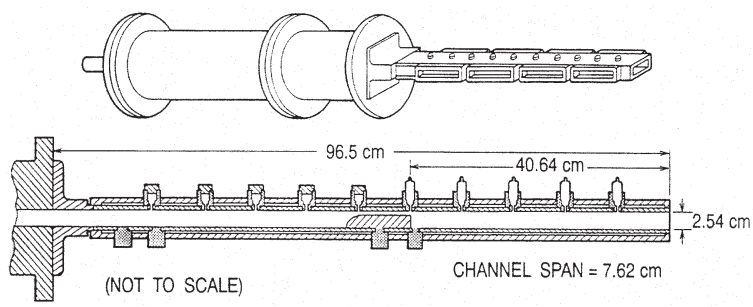
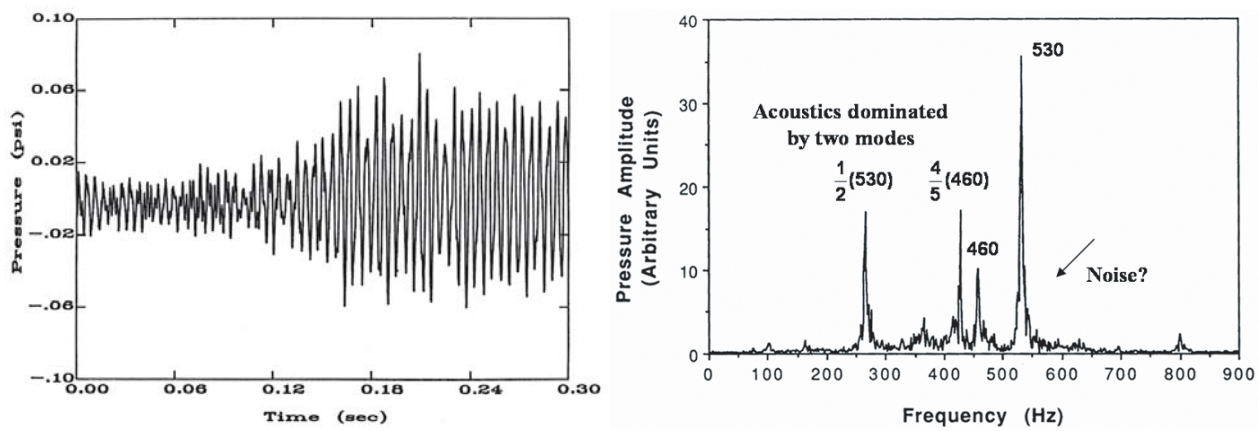


FIGURE 1.17. The Caltech dump combustor.

oscillations having frequencies 530 Hz. and 460 Hz. and subharmonics. Estimates based on the assumption of axial acoustic motions have shown (Sterling 1987; Sterling and Zukoski 1987, 1991; Zsak 1993) that the two oscillations are normal modes of the system. Explanation of the nonlinear mechanism responsible for the sub-harmonics has not been given. It is interesting and significant that the 'noise' exhibited in the spectrum seems to appear as a kind of random modulation of the amplitude of the waveform reproduced in Figure 1.18(a). That interpretation is supported by the approximate analysis of nonlinear acoustics and noise covered here in Chapter 7.



(a) (b)

FIGURE 1.18. Waveform and spectrum for an instability in the Caltech dump combustor.

Although this example is special, it does illustrate the chief features of combustion instabilities generally: well-defined organized oscillations within an apparently random field, normally called noise. It is quite common that there are more peaks in the spectra than appear in Figure 1.18(b), and that the frequencies tend to be close to those of the normal acoustics modes of the chamber in question. The quantitative aspects vary widely, but the physical behavior suggested by these results broadly defines the general problem to be addressed by the theory.

The dump combustor has been a favored configuration for laboratory tests having a variety of forms and purposes. We will appeal often to results of such tests. Particularly, the configuration is prone to generate large vortices generated in various forms, a widely studied phenomenon. The dump combustor continues to be a favorite configuration for research as well as applications. Early work by Schadow and his colleagues has been extended by many followers. (Schadow *et al.* 1981, 1983, 1984, 1985, 1987a, b)

1.2.4. Combustion Instabilities in Gas Turbines. Until fairly recently, combustion instabilities in gas turbines had received much less attention than in other systems. We distinguish here main combustors from afterburners which have always exhibited troublesome oscillations. Traditionally, and it is still true, relatively little information has been available about such instabilities in practical systems, for proprietary reasons. However it is probably true for several reasons, mainly relatively large acoustic losses in the combustors, that until fairly recently serious combustion instabilities had been quite rare in gas turbines.

The situation changed with increased emphasis on reducing air pollution. At about the same time, the use of gas turbines for stationary power generation increased. Thus serious concern with any development problems also increased and was not so restricted by proprietary considerations owing to the widespread implications. A strategy for reducing emission of a major pollutant, NO, is to lower the average temperature at which primary combustion occurs, in accordance with the Zel'dovich mechanism for producing NO (Zel'dovich *et al.* 1985). Unfortunately, at lower temperatures achieved by operation at lower local values of fuel/air ratio, the processes stabilizing the flame are less stable and tend to encourage the excitation of oscillations.

As a result, during the past ten–twelve years combustion instabilities have become a serious problem in the development of stationary power generation systems based on combustion, mainly of hydrocarbon fuels. That is not a subject central to this book but the problems are in some cases similar and the methods discussed here are applicable. The subject of combustion instabilities in gas turbines has become important for very practical reasons, but basic problems remain unsolved. Chapter 9 is an abbreviated discussion of instabilities and their active control in gas turbines.

1.2.5. Combustion Instabilities in Solid Propellant Rocket Motors. Instabilities in solid propellant rockets were the first examples discovered, as early as the late 1930s. For reasons which will become clear in this book, unsteady motions in full-scale solid propellant motors and in laboratory devices have probably been the subject of more research than in any other type of system. Accordingly, much of our approach to understanding instabilities in combustion chambers generally can be traced to experiences with solid rockets. An excellent authoritative summary of the research and practical sides of the subject is the short historical article by Price (1992).

Since the late 1950s, serious concern with instabilities in solid propellant motors has been sustained by problems arising in both small (tactical) and large (strategic and large launch systems) rockets. The volume of collected papers compiled and edited by Berle (1960) provides a good view of the state of the field at the end of the 1950s in Western countries. The level of activity remained high and roughly unchanged through the 1960s, due entirely to the demands of the Cold War: The use of solid rocket boosters in systems for launching spacecraft, and for changing trajectories, came later. During the 1950s and 1960s strong emphasis was already placed on sub-scale and laboratory tests, a strategy dictated at least partly by the large costs of full-scale tests. As a result, more is understood about combustion instabilities in solid rockets than for other systems. Moreover, methods and viewpoints developed by the solid rocket community have strongly influenced the approaches to treating combustion instabilities in other systems. The theory developed in this book is an example of that trend.

Many earlier cases exist of combustion instabilities in solid rockets, but a particularly striking example arose in the late 1960s and was documented in the AIAA/ASME Joint Propulsion Meeting in 1971. It was a problem with the third stage of the Minuteman II launch vehicle that initially motivated considerable research activity during the following decade, sponsored largely by the Air Force Rocket Propulsion Laboratory. The causes of three failures in test flights had been traced to the presence of combustion instabilities. Thorough investigation showed that although oscillations had been present throughout the history of the motor, a significant change occurred during production, apparently associated with propellant Lot 10. A record of

the pressure during a flight test is given in Figure 1.19. The broadening of the trace is due to the presence of 'high' frequency oscillations, close to 500 Hz. Figure 1.20 shows the main observable features.

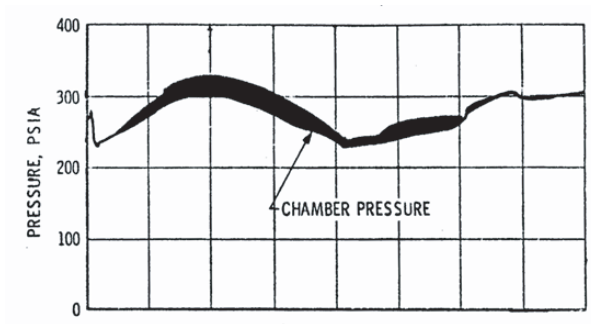


FIGURE 1.19. Flight test record of pressure in a Minuteman II, stage 3.

The oscillations existed during the first fifteen seconds of every firing and always had frequency around 500 Hertz. Whatever had occurred with production Lot 10 caused the maximum amplitudes of oscillations to be unpredictably larger in motors containing propellant from that and subsequent lots. The associated structural vibrations caused failures of a component in the thrust control system.

This example exhibits several characteristics common to many instances of combustion instabilities in solid rockets. In test-to-test comparisons, frequencies are reproducible and amplitudes show only slight variations unless some change occurs in the motor. Any changes must be of two sorts: either geometrical, i.e. the internal shape of the grain; or chemical, consequences of variations in the propellant. Chemical changes, i.e. small variations in the propellant composition, are most likely to affect the dynamics of the combustion processes and indirectly other physical processes in the motor. That is apparently what happened in the Minuteman.

Between production of propellant Lots 9 and 10, the supplier of aluminum particles was changed, because the original production facility was accidentally destroyed. The new aluminum differed in two respects: shapes of the particles, and the proportion of oxide coating on individual particles. Testing during investigation of the instability led to the conclusion that consequent changes in the processes responsible for the production of aluminum oxide products of combustion generated smaller particle sizes of Al_2O_3. The smaller sizes less effectively attenuated acoustic waves; the net tendency to excite waves therefore increased. As a result, the motors were evidently more unstable and also supported larger amplitudes of oscillation. The second conclusion was purely speculative at the time of the investigation, but can now be demonstrated with the theory covered in this book. Nevertheless, the details explaining why the change in the aluminum supplied led eventually to the significant changes in the combustion products remain unknown.

Subsequent to the Minuteman problem, the Air Force Rocket Propulsion Laboratory supported a substantial program of research on many of the most important problems related to combustion instabilities in solid rockets. Broadly, the intellectual centroid of that program lay closer to the areas of combustor dynamics and combustion dynamics than to the detailed behavior of propellants. The synthesis, chemistry, and kinetics of known and new materials belonged to programs funded by other agencies in the U.S. and in Europe, notably ONERA in France. By far most of the related work in Russia has always been concerned with the characteristics and combustion of propellants, with relatively little attention to the dynamics of combustors.

Investigations and problems of combustion instability in solid rockets have been far more numerous and widespread than for any other system. It is likely an accurate statement that more is generally understood about the problem of oscillations in combustors because of work done to treat the problem in solid rockets.

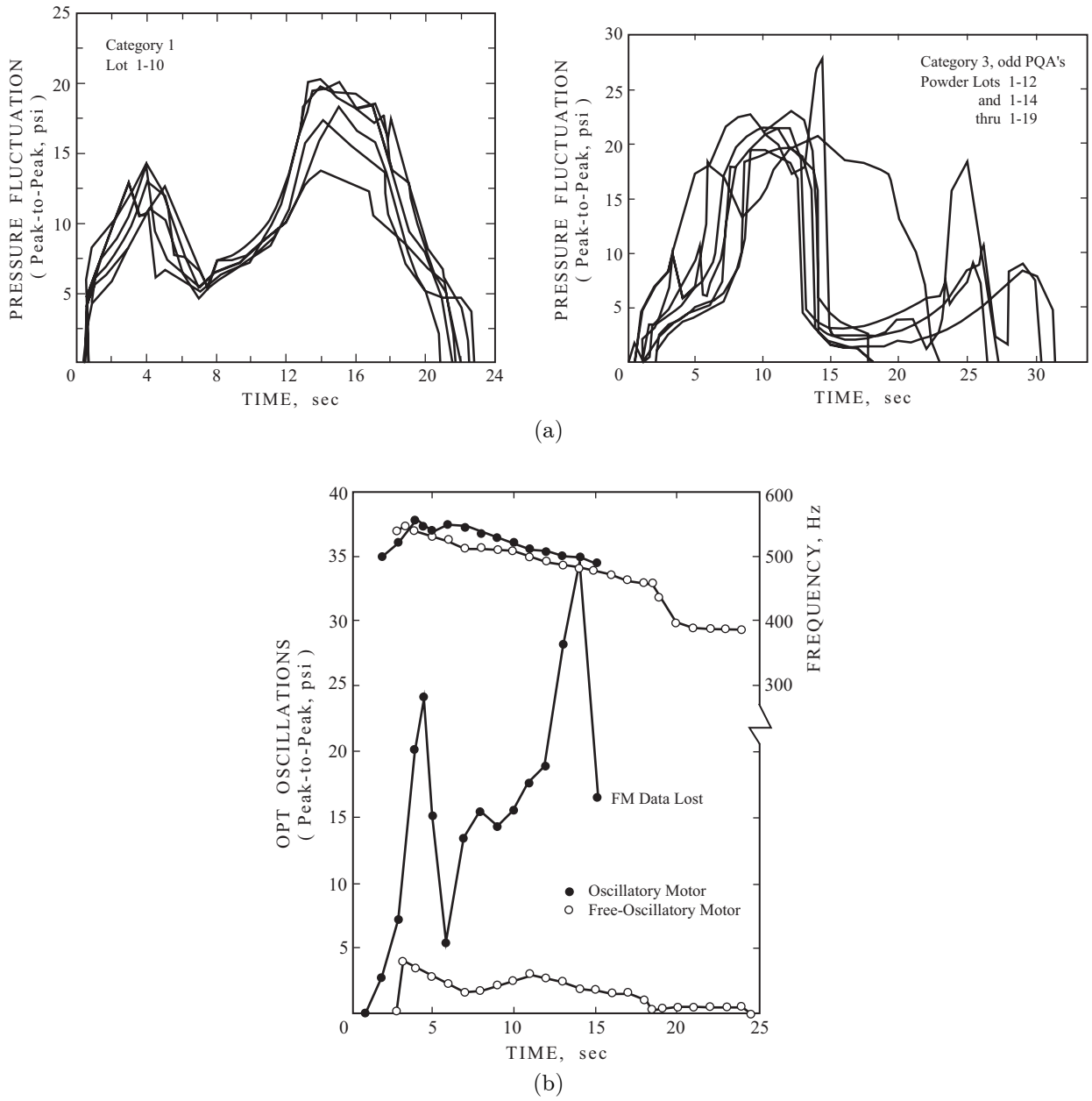


FIGURE 1.20. Frequencies and amplitudes of combustion instabilities in the Minuteman II, stage 3 motor: (a) change of behavior after Lot 1-10 (Fowler and Rosenthal 1971); (b) frequencies and amplitudes measured during static tests (Bergman and Jessen 1971).

We will mention only a few examples here and refer to others where appropriate in the following text. As Figure 1.2 suggests, much had been accomplished prior to the Minuteman problem. A useful selection of papers was given in a session at the AIAA/ASME Joint Propulsion Meeting, 1971.

In the late 1960s and 1970s there was considerable effort in the U.S. to "solve" the problem of instabilities in solid rockets. Similar research was carried out in Europe, especially in France. Considerable progress

was made, both theoretically—notably the foundation of a method for analyzing nonlinear behavior—and experimentally, especially methods to measure the response function of solid propellants improved at several U.S. companies, at the Naval Air Warfare Center, China Labs (NAWC) and at ONERA in France. But problems of instabilities in motors continued to arise without prediction.

At that time it was known that there remained some difficulties with the theory and serious computational restrictions. That is true as well today although progress continues to be made. The most recent large research program devoted to the subject in the U.S. was the Multidisciplinary University Research Initiative (MURI), "Investigations of Novel Energetic Materials to Stabilize Rocket Motors." Although the research was primarily concerned with properties and behavior of energetic propellants, some of the work necessarily was devoted to general problems of time-dependent combustion. One conclusion, confirming once again previous work, is that there is no method giving accurate and widely useful results for the propellant response function. An idea of how difficult the problem is may be gained from the fact that at least five methods have been used, several extensively, and none has been found satisfactory for all purposes.

There seems to be general agreement that for most practical purposes the T-burner serves best. This device, sketched in Figure 1.21, in principle gives data for the response of a small flat sample of propellant to a pressure oscillation. Variants have been devised to give results for the response to velocity fluctuations, with only modest success. First discussed by Price and Sofferis (1958), by far most work with the T-burner has been done at NAWC. Price (1992) has given the most authoritative summary of instabilities in solid rockets, including the development of T-burners. The most recent discussions of T-burners have been given by Blomshield *et al.* (1996) and Blomshield (2000).

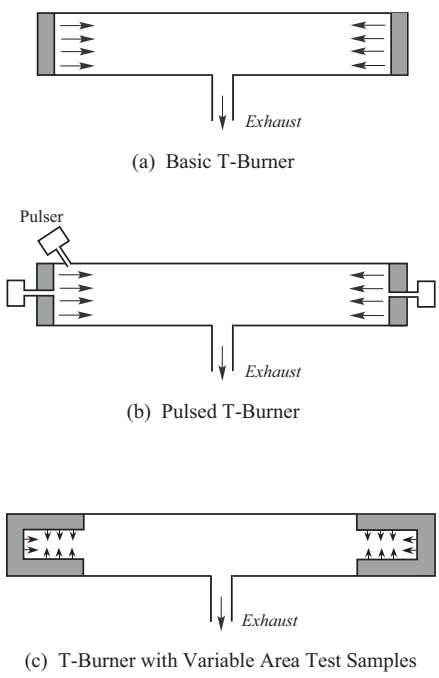


FIGURE 1.21. A sketch of the basic T-burner and two variants.

Blomshield (2001) has also compiled a large list of examples of instabilities in operational motors. Figure 1.22 taken from that work is an impressive array of motors that have exhibited instabilities. The examples cover more than 45 years.

Motor Details					Type				Fix			
No.	Name	Date	App.	Propellant Type	Long	Tang	DC shifts	Pulsed	Prop	Metal	Geo.	Details
1	Sergeant	1951	Sounding	Polysulfide/AP		X						Not fixed
2	RVA-10	1951	TBM	Polysulfide/AP		X						Unknown
3	Sergeant	1957	TBM	Polysulfide/AP		X						Not Fixed
4	Subroc	1961	SAS	Al/Polyurethane	X		X			X		Reduced Al size
5	Iroquois	1960	?	Aluminized	X		X			X		Increased Al%, reduced Size
6	Tartar	1961	SAA	Duel Grain	X		X	X			X	Nozzle moved downstream
7	Tow	1964	STS	Double Base		X	X				X	Added Baffles
8	Genie	1965	ATA	AP/Al/Polyurethane	X					X		Reduced Al size
9	Minuteman	1968	BAL	Double Base/AP/Al	X					X		Changed system, not motor
10	Manpads	1969	SL	AP/Al/HTPB	X		X		X			Lowered Solid Loading
11	ATR	1975	ATA	AP/HTPB		X	X		X			Increased AP size, added catalyst
12	AALM	1975	RES	AP/HTPB				X	X			ZrC containing motors were better
13	MK-12	1975	SAA	AP/Al/HTPB	X	X						Changed system, not motor
14	Slufae	1975	STS	AP/HTPB	X	X					X	Added helmholtz resonator
15	Sidewinder	1977	ATA	AP/HTPB	X		X	X	X		X	Grain design change, some RDX for AP
16	Maverick	1977	AS	AP/HTPB		X	X		X			Increased AP size, added catalyst, ZrC
17	LCMM	1978	RES	AP/HTPB	X		X				X	Add eroding Nozzle, changed geometry
18	MSM	1978	RES	Double Base/CMDB		X	X				X	Increased port area
19	Harm	1978	AS	AP/HTPB	X						X	No changes required
20	EX-70	1979	SAA	AP/HTPB	X				X		X	Smaller AP, increase nozzle size
21	EX-104	1985	SAA	Duel AP/Al/HTPB	X							No changes required
22	ASROC	1985	SAS	AP/HTPB	X							No changes required
23	Sentry	1985	TBM	AP/Al/HTPB	X		X			X		Program ended, smaller Al
24	LCPM	1988	???	AP/HTPB		X			X			Smaller AP, higher loading
25	SHUTTLE	1990	BOS	AP/Al/PBAN	X							No changed required
26	DBM	1994	SL	AP/Al/HTPB	X		X				X	Grain design change
27	Pathfinder	1996	SPA	AP/Al/HTPB	X				X	X		Increased Al from 2% to 16%
28	NWR	1997	RES	AP/HTPB	X	X	X	X	X		X	Stability additives, geometry, pressure

TBM – Theater Ballistic Missile SAS – Surface Anti-Submarine SAA – Ship Launched Anti-Aircraft STS – Surface to Surface SPA – Space Motor
ATA – Air-to-Air BAL – Ballistic Missile SL – Surface Launched AS – Air to Surface RES – Research Motor BOS – Space Booster

FIGURE 1.22. A partial summary of U.S. solid propellant motors (1951–1997) having problems of combustion instabilities (Blomshield 2001).

Not covered in the summary just cited is the case which has generated more effort than any other in the recent past—the Ariane 5 Booster. Both the Shuttle and the Ariane 5 booster motors exhibited relatively low level longitudinal oscillations. However, the excursions of pressure in the Ariane 5 have consistently been larger and due to installations of counter-measures, may have cost payload.[4] The problem has motivated a large amount of very good research and has led to the discovery of a new source of pressure waves called "parietal vortex shedding" discussed further in the following section. Whereas the sort of vortex shedding already mentioned, and commonly found in combustors, occurs at edges, parietal vortex shedding arises when vortices are formed near a boundary through which flow enters a volume. Computations and several tests in laboratory devices of established reality of the phenomenon. It appears, however, that disagreement still exists concerning the importance of parietal vortex shedding in the full-scale Ariane 5.

As the research activities related specifically to solid rockets decreased during the 1980s and new programs began for liquid-fueled systems, the communities, previously quite separate, grew closer together.

[4]As part of the vehicle design—before the first tests—mechanical damping devices were installed between the booster motors and the main vehicle. The problem is particularly bothersome in practice because for odd acoustic modes, there is a large amplification factor (about ten for the Ariane 5) relating fractional thrust oscillations to fractional pressure oscillations. (P. Kuentzmann, private communication.)

For example, prompted by contemporary concern with problems in ramjets, a workshop sponsored by JANNAF (Culick 1980) was organized partly with the specific intention to bring together people experienced in the various propulsion systems. During the 1980s there was considerable interchange between the various research communities and since that time, a significant number of people have worked on both solid and liquid-fueled systems. That shift in the sociology of the field has provided the possibility and much of the justification for this book. Events of the past decade have confirmed that the field of combustion instabilities is very usefully approached as a unification of the problems arising in all systems.

In Europe during the 1990s, work on combustion instabilities in solid propellant rockets was motivated largely by low frequency oscillations in the booster motors for the Ariane 5. The most intensive and comprehensive recent work in the U.S. has been carried out in two Multidisciplinary University Research Initiatives (MURI) involving 15 different universities (Culick 2002a, Krier and Hafenrichter 2002). An unusual characteristic of those programs, active for six years beginning in 1995, was the inclusion of coordinated research on all aspects of problems of combustion instabilities in solid propellant rockets, from fundamental chemistry to the internal dynamics of motors. Results of recent works will be covered here in the appropriate places.

1.3. Mechanisms of Combustion Instabilities

The simplest and most convenient characterization of an unstable oscillation is expressed in terms of the mechanical energy of the motion. Linear theory produces the result that the rate of growth of the amplitude is proportional to the fractional rate of change of energy, the sum of kinetic and potential energies. The idea is discussed further in the following section. What matters at this point is that the term 'mechanism' refers to a process that causes transfer of energy to the unsteady motion from some other source. Thus, mechanisms form the substance of the feedback path in Figure 1.1. Generally there are only three sorts of energy sources for unsteady motions in a combustor: the combustion processes; the mean flow, which of course itself is caused by combustion; and a combination of combustion and mean flow simultaneously acting. The distinction is important because the physical explanations of the energy transfer are very different in the three cases.

Just as for steady operation, the chief distinctions among combustion instabilities in different combustors must ultimately be traceable to differences in geometry and the states of the reactants. The root causes, or 'mechanisms', of instabilities are imbedded in that context and are often very difficult to identify with certainty. Possibly the most difficult problem in any particular case is to quantify the mechanism. Solving that problem requires finding an accurate representation of the relevant dynamics.

1.3.1. Mechanisms in Liquid-Fueled Rockets. Combustion instabilities first became a serious problem in liquid rockets and remain a matter of basic concern during development. The chief mechanisms remain those known for many years, associated with the propellant feed system; the injection system; the processes required for conversion from liquid to gas; and combustion dynamics. There seem to be no examples caused primarily by vortex shedding, mean flow/acoustics interactions, or convective waves (of entropy or vorticity).

Figure 1.23 is a broad summary of the main processes taking place in a liquid rocket combustion chamber (Culick and Yang 1993) The listing and categorization serves only as a rough general guide. Particular situations may introduce additional processes.

In order to construct a dynamical model of a combustion chamber, it is necessary to place the processes of Figure 1.23 in space. This is not the occasion to pursue that matter in detail, which must in any case be done for each given design. Figure 1.24, adapted from Bazarov (1979), conveys the general idea. One approach to analyzing linear stability is based on combining the transfer functions according to a diagram like Figure 1.24. Then the problem may be posed in the manner of feedback control theory. Each process

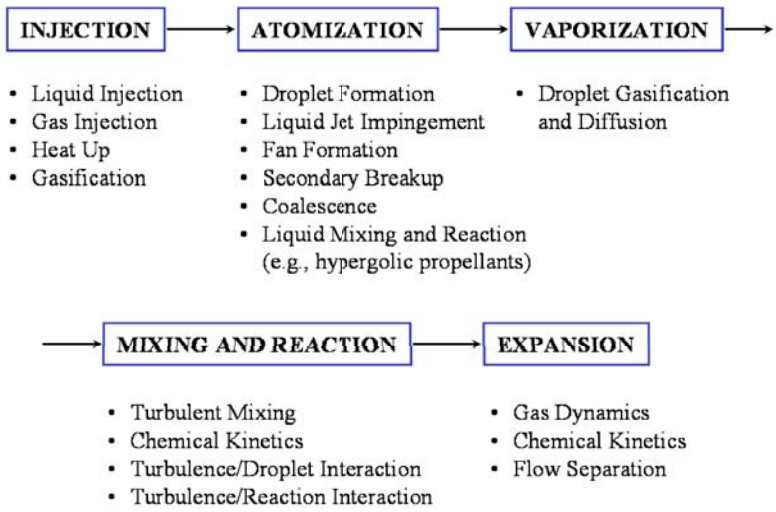


FIGURE 1.23. An approximate schematic summary of the important rate processes in a liquid rocket combustor (Culick and Yang 1993).

suggested or alluded to in Figure 1.23 must then somehow be represented by a transfer function. That may be a difficult and complicated task particularly due to the coupling between the processes.

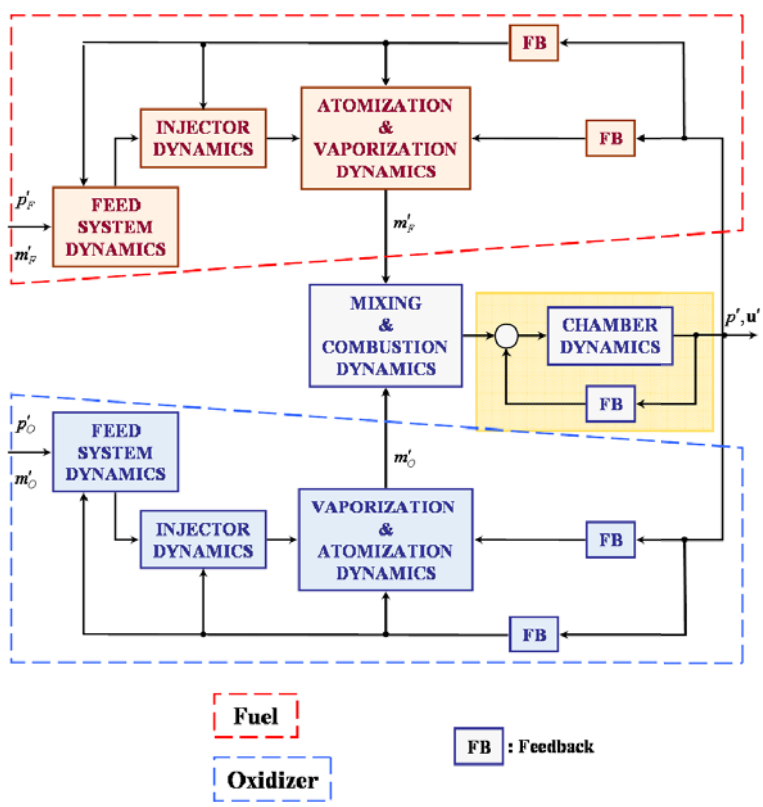


FIGURE 1.24. A simplified diagram for the dynamics of a liquid rocket engine (adapted from Bazarov 1979).

The mechanisms causing instabilities differ in detail, and perhaps grossly, depending on the type of system. Generally there are three types of system depending on the choice of reactants: liquid oxygen/hydrogen (LOX/H) typified by the engines RL-10, J-2, Space Shuttle main engine (SSME), and the Vulcain (European Space Agency); liquid oxygen/hydrocarbon (LOX/HC) of which some examples are the Apollo F-1, the Atlas, and the RD-0110 (Russian); and those systems using storable propellants such as those based on nitrogen tetroxide (N_2O_4) as the oxidizer with fuels commonly hydrazine, monomethylhydrazine (MMH) as in the French Viking motor in the Ariane 1-4, and unsymmetrical dimethylhydrazine (UDMH). The TRW pintle engine which was used in the Lunar Descent Vehicle is perhaps the most famous engine using storable propellants.

For a given choice of reactants, the most significant influence on the instabilities is the injection system. In this context the main classes of injectors are impinging jets; shear and swirl coaxial injectors; showerheads; and impinging sheets. Changes of the geometry may produce a large variety of injectors, many of which are shown in Figure 1.5. Each of these devices exhibits its own mechanism for instability, possibly different in important respects, and subtly dependent on operating conditions. It is not possible to offer generalizations. Figure 1.25 suggests the variety of mechanisms that may arise in a coaxial injector.

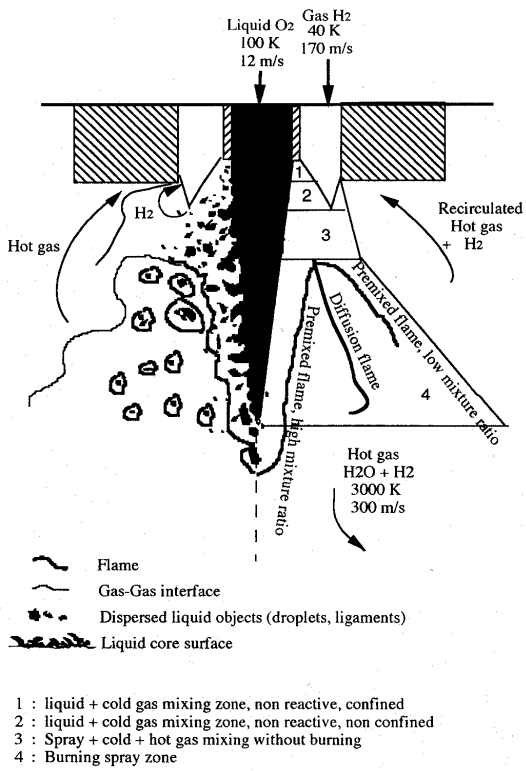


FIGURE 1.25. A sketch showing some of the processes taking place in a coaxial injector (Vingert *et al.* 1993).

1.3.2. Sensitivity of Combustion Processes; Time Lags. Combustion processes are sensitive to the macroscopic flow variables, particularly pressure, temperature and velocity. Even slow changes of those quantities affect the energy released according to rules that can be deduced from the behavior for steady combustion. In general, however, representations of that sort, based on assuming quasi-steady behavior, are inadequate. Combustion instabilities normally occur in frequency ranges such that genuine dynamical

behavior is significant. That is, the transient changes of energy release do not follow precisely in phase with imposed changes of a flow variable such as pressure.

The simplest assumption is that the combustion processes behave as a first order dynamical system characterized by a single time delay or relaxation time. That idea was apparently first suggested by von Karman as a basis for interpreting instabilities discovered in early experiments with liquid propellant rockets at Caltech (Summerfield 1941). That representation, which came to be called the '$n-\tau$ model' was developed most extensively by Crocco and his students at Princeton during the 1950s and 1960s. Time delays may be due, for example, to processes associated with ignition of reactants. Subsequent to injection as the reactants flow downstream, finite times may be required for vaporization, mixing, and for the kinetics mechanism to reach completions. Both effects may be interpreted as a convective time delay. Under unsteady conditions, the initial state of the reactants, their concentrations, pressure, and velocity, also fluctuate, causing the delay time to be both nonuniform in space and in time. As a result, the rate of energy release downstream in the chamber is also space- and time-dependent, and acts as a source of waves in the combustor.

In the case of liquid-fueled systems, interactions of the injected streams, formation of sheets and break-up into drops are processes sensitive to pressure and particularly velocity fluctuations. Those are purely fluid-mechanical processes impossible to treat analytically and pose extremely difficult problems for numerical simulations. No complete numerical analyses exist and only much simplified models have been used in numerical simulations of combustion instabilities. The dynamics of a combustion system are not likely to be well-represented by an $n-\tau$ model.

If a time-lag model is used, either further modeling and calculations must be carried out, or an assumption must be made for the dependence of the time lag on frequency. It is usual that the time lag is assumed constant. Then the response of the system exhibits unrealistic resonances as the frequency increases. An example is shown later in Figure 2.20 for combustion of a solid propellant. Realistic behavior is found only by taking the lag itself to be a function of frequency. The choice is largely arbitrary unless the physical basis for the model is improved.

Purely gaseous-fueled systems present possibilities for different physical models that also leads to first-order behavior. It is an old idea that even in complicated geometries, combustion in a non-premixed system must occur at least partly in elements of diffusion flames. If the gaseous reactants are premixed then in simple configurations such as tubes, or dump combustors, combustion may occur in large stable flame sheets of the flow is laminar, or in fragments of premixed 'flamelets' when the flow is turbulent. In all of these cases it is reasonable to anticipate that at any given time the rate of energy release is roughly proportional to the area of the flame sheets. Then fluctuations of the velocity or processes responsible for ignition and extinction, will cause fluctuations of the energy release rate. Models of this process lead to an equation representing first order behavior (For example, see Poinsot *et al.* 1988; Candel *et al.* 1992; McManus *et al.* 1993; Dowling 1995; Annaswamy *et al.* 2000).

The approximation of first order behavior fails entirely for the dynamics of burning solid propellants (Culick 1968). Although in good first approximation dominated by unsteady heat transfer in the condensed phase, a diffusive process, the combustion dynamics in this case exhibits behavior closer to that of a second order system. The frequency response of that burning rate tends normally to have a large broad peak centered at a frequency falling well within the range of the frequencies characteristic of the chamber dynamics. Hence there is a clear possibility for a resonance and instability suggested by the diagram in Figure 1.1.

1.3.3. Vortex Shedding and Vortex/Mean Flow Interactions. Generation of oscillations by the average flow is due to causes roughly like those active in wind musical instruments. In all such cases, flow separation is involved, followed by instability of a shear layer and formation of vortices. Direct coupling between the vortices and a local velocity fluctuation associated with an acoustic field is relatively weak; that

is, the rate of energy exchange is in some sense small. However, the interaction between the velocity (or pressure) fluctuation and the initial portion of the shear layer is normally a basic reason that feedback exists between the unsteady field in the volume of the combustor and vortex shedding.

Figure 1.26 conveys the idea of one way in which shed vortices may excite oscillations. The local acoustic velocity field determines the times at which individual vortices are shed at an edge. By some means of coupling, the vortices then act to generate an acoustic field, but with a lag between the shedding and generation processes. The acoustic field so generated is local and the process of filling the chamber is another matter. However, it seems to be generally true that acoustic fields generated by vortex shedding are never as intense as the stronger fields produced by the combustion processes directly.

Byrne (1981, 1983) seems to have been first to suggest that vortex shedding could be responsible for instabilities observed in dump combustors. The idea was developed very actively in the 1980s, both in systems operating at room temperature without combustion (Schadow *et al.* 1981, 1983, 1984, 1985) and in combustors (Schadow *et al.* 1985, 1987a, 1987b).

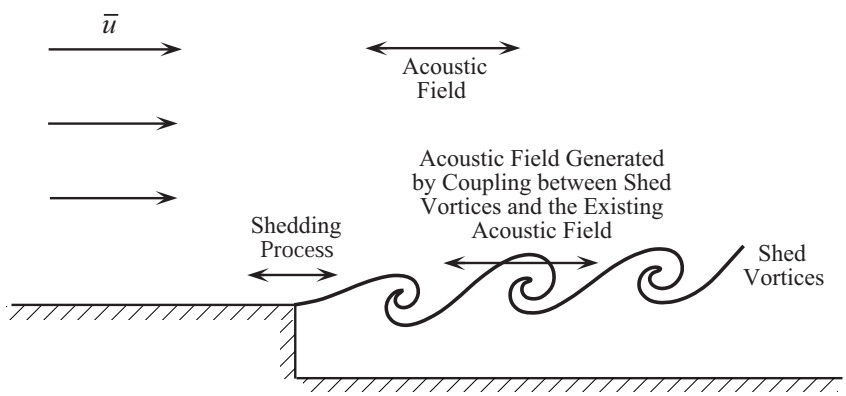


FIGURE 1.26. Generation of an acoustic field by vortex shedding.

Unsteady combustion in vortices was one of the early mechanisms proposed as a cause of combustion instabilities in combustors using bluff body flameholders (Kaskan and Noreen 1955, Marble and Rogers 1956). It was essentially re-discovered in the 1980s during tests of dump combustors (Smith and Zukoski 1985; Daily and Oppenheim 1986; Sterling and Zukoski 1991, for example). Several attempts have been made to quantify the mechanism with analysis (Norton 1983; Karagozian and Marble 1986) and with numerical simulations (Laverdant and Candel 1988; Samaniego and Mantel 1999). Insufficient progress has been made to construct a model suitable for general analysis of combustion instabilities. Thus there is currently no basis for predictions of combustor dynamics excited by this mechanism, although there are recent simplified attempts for special situations by Matveev and Culick (2002, 2003a) and by Matveev (2004) . Special cases have been discussed in connection with particular experimental results; see, for example, Sterling 1993 and Sections 2.3.4 and 2.3.5.

It has long been known experimentally that vortices shed in a chamber more effectively generate acoustic waves if they impinge in an obstacle downstream of their origin (Flandro and Jacobs, 1975; Culick and Magiawala, 1979; Nomoto and Culick, 1982; Flandro, 1986). The first practical examples of this phenomenon were found in the solid rocket booster for the Shuttle launch system in the 1970s and the Titan motor (Brown *et al.* 1981, 1985). It was that problem that motivated the works just cited, but since then vortex shedding has been recognized as a mechanism for generating acoustic oscillations in other systems as well, notably the booster motors on the Space Shuttle and on the Ariane 5. A particularly good summary and discussion of the subject has recently been given by Vuillot and Casalis (2002). Owing to their involvement with a problem of pressure oscillations in the Ariane 5 booster motors, the authors focus special attention on a very

different form of vortex shedding. Laboratory tests and numerical simulations have established the existence of a process called 'parietal vortex shedding' in which large coherent vortices grow out of the region of shear near a surface through which flow enters the chamber. The flow can either be issuing from a burning solid propellant or may be the flow inward of fluid through a permeable boundary.

It happens that the amplitudes of oscillations in the Ariane motors are significantly greater than those found in the Space Shuttle booster motors. In the latter case, vortices are definitely created by instability of the shear layer formed in flow past an obstacle. The reasons for the difference in amplitudes in the two cases may be due to differences in geometry. One proposal rests on special conditions in the Ariane, possibly encouraging "parietal vortex shedding", but the problem is not satisfactorily resolved. Parietal vortex shedding involves growth of vortices initiated in the region adjacent to a transpiring surface, such as a burning solid.

1.3.4. Operation Near the Lean Blow-Out Limit. Combustion instabilities have not historically been a serious problem in gas turbine main combustors. Although instabilities have certainly been observed for many years, they have not been persistently troublesome. Due to proprietary considerations almost no detailed results for full-scale machines have been made public, a situation that has recently been changing. In the past few years combustion instabilities have become a serious problem in gas turbines because of the need to operate close to the lean blowout limit of premixed gaseous reactants as part of the strategy to reduce generation of pollutants, notably NO_x.

As the operating condition approaches the lean blowout limit, combustion processes ('flame dynamics'), including flame stabilization, are more sensitive to fluctuations than under operation at higher mixture (F/O) ratios. The sensitivity extends to flame fronts and zones as well as to the stabilization processes, shear layers and recirculation zones. The latter, associated with injection and stabilization, may possess multiple dynamical states, i.e. special bifurcations and hysteresis.

The dynamical behavior of the premixer and injection devices may contribute to instabilities in various ways. Internal resonances, for example, may be excited by oscillations in the chamber, causing perturbations of the energy released in the combustion processes downstream of the injector. There may also be undesirable coupling between elements of an array of premixers and injectors. Such dynamical behavior may also be turned to advantage to extend the operating range of stable operation. That strategy was successfully pursued on several occasions in the Russian liquid rocket community.

It is likely that fluctuations of the mixture or fuel/oxidizer ratio (F/O) play an important role in the dynamics of gas turbine combustors (Lieuwen *et al.* 2001). If the F/O ratio of the reactants is at all sensitive to conditions in the combustion chamber, there is an obvious feedback path connecting the combustor and combustion dynamics. The possibility has arisen previously in other combustion systems, but at least anecdotal evidence has suggested that serious attention must be paid to fluctuations of mixture ratio as a fundamental mechanism for instabilities in gas turbines.

There is considerable evidence that the ratio of a "convective timescale" and the dominant acoustic period (1/f) may be the determining parameters, or at least an important one, governing mixture ratio fluctuations in a given situation. Lieuwen *et al.* (2001) and Cohen *et al.* (2003) have given particularly useful discussions of the matter. Unfortunately the situation is clouded because the first case, shown in Figure 1.27, is a discussion of an axisymmetric configuration and the second, Figure 1.28, is concerned with experiments using a two-dimensional dump combustor. The ratio $\tau_c f$ was around unity or less for the tests with the configuration shown in Figure 1.27, but two or greater for the two-dimensional combustor, Figure 1.28. It seems that explanations of the observed behavior are more convincing when considered separately than when taken together. There is no single reasoning, no matter how lengthy, that explains satisfactorily the observations. The actual case, like the others treated here, is too complicated for such a simple result.

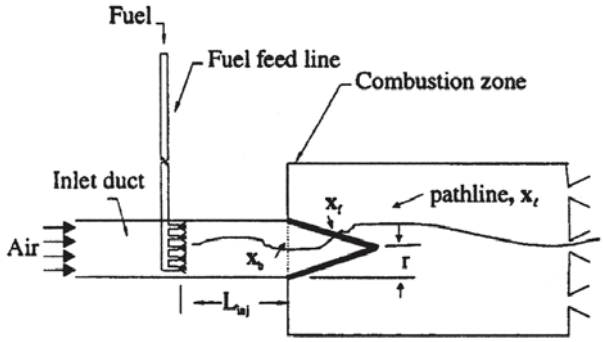


FIGURE 1.27. A form of axisymmetric lean premixed combustor (Lieuwen *et al.* 2001).

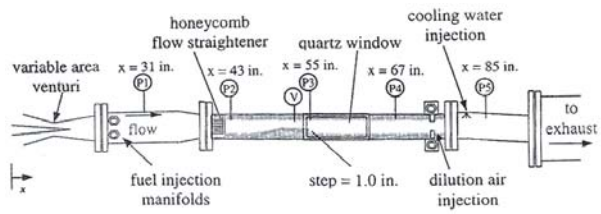


FIGURE 1.28. A typical 2-D dump combustor (Cohen *et al.* 2003).

Much remains to be learned about instabilities in gas turbines, although the subject has been greatly clarified in the past 10–12 years. We will not cover the material thoroughly in this volume, mainly because we are concerned primarily with other types of systems which are of more concern for the applications most significant for aerospace systems. Moreover, the causes of instabilities in gas turbines to a large extent seem to be distinct from those in rockets, afterburners and ramjets. In the most fundamental sense, that conclusion is perhaps illusory. Chapter 9 is devoted to a largely qualitative discussion of some of the recent topics addressed in work on instabilities in gas turbines.

1.3.5. Mechanisms in Solid-Fueled Rockets. No disagreement exists that the predominant mechanisms for instabilities in solid propellant rockets is the sensitivity of burning surfaces to pressure and velocity fluctuations. A large part of Chapter 2 is devoted to the basic essentials, including, in Section 2.2, a detailed derivation of the simplest response of a burning surface to pressure fluctuations. During the past five decades, a great amount of resources has been consumed in an effort to develop methods for measuring the response. Success, however, has been spotty and far short of what is required both for practical and for research purposes. It is an outstanding problem in the field that merits continued work, especially to devise a new method.

Quite a different mechanism also exists, involving the shedding of large vortices and subsequent excitation and interactions with acoustic waves. Vorticity/acoustic coupling is actually a relatively widely recognized mechanism, not only in solid rockets, because its existence does not rest on combustion processes. The importance of vortex shedding in dump combustors, the usual configuration of ramjet engines, has long been established as a major mechanism. Thus, if one is concerned with the possibility that the process is important in a solid propellant motor, attention should be paid to the characteristics of the phenomenon in ramjets. The point is not, of course, that the two sorts of situations are in some sense the 'same' but there are similarities and much may be learned from comparisons of results. No such comparative studies exist.

Partly because a solid propellant rocket cannot be repeatedly tested in the same manner as other systems, more attention has been given combustion instabilities in this kind of combustor than for any other. It is possible, but not likely, that any important mechanisms of instabilities have been overlooked or are unknown. The practical problem, as we will see, is quantifying the contributions well enough to obtain good results.

1.3.6. Combustion Dynamics. Except for instabilities sustained by the purely fluid mechanical mechanism of vortex shedding, all practical cases of combustion instabilities involve combustion dynamics in some form. It is hardly an exaggeration to claim that understanding combustion dynamics is ultimately the most important fundamental problem in the subject of combustion instabilities. Broadly the term refers to the

property possessed by any chemically reacting system that the rate at which energy is released is sensitive, i.e. responds, to the rates of change of pressure, temperature, density and mixture ratio. For solid and liquid propellants, the combustion dynamics is usually more conveniently expressed in terms of the rate of conversion of the condensed phase to gaseous combustion products.

In the limit of small changes (i.e. for linear behavior), dynamical response functions for combustion systems are entirely analogous to transfer functions used in the subject of controls. They are fundamental to analysis of the stability of combustion systems and to application of the principles of feedback control.

Introduction of response functions as general definitions is part of the basis on which combustion instabilities, or, more widely, the dynamics of combustion systems, can be investigated and understood with minimal appeal to the particulars of the combustion dynamics of a specific type of system. However, applications of the theory require close attention to the details of the system at hand. Modeling combustion dynamics on theoretical grounds only carries serious limitations due to the complexities of the systems. Hence experimental methods and, to an increasing extent, numerical simulations are essential to treating combustion instabilities in practical systems.

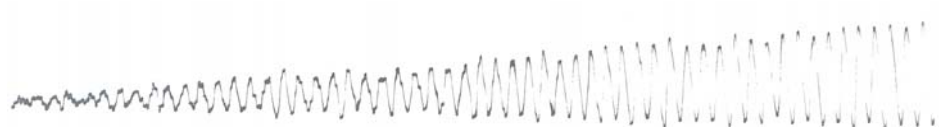
1.4. Physical Characteristics of Combustion Instabilities

Owing to the difficulty of making direct measurements of the flow field within a combustion chamber, virtually all that is known about combustion instabilities rests on close coordination of experiment and theory. The subject is intrinsically semi-empirical, theoretical work being founded on observational data both from full-scale machines and laboratory devices. Conversely, the theoretical and analytical framework occupies a central position as the vehicle for planning experimental work and for interpreting the results. The chief purposes of this section are to summarize briefly the most important basic characteristics of observed instabilities; and to introduce the way in which those observations motivate the formulation of the theoretical framework.

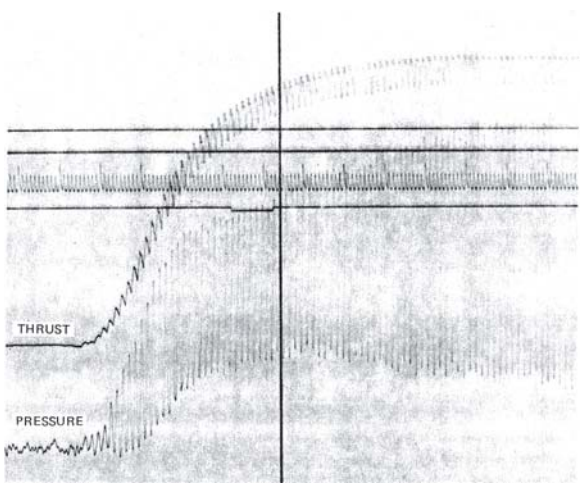
In tests of full-scale propulsion systems, only three types of data are normally available, obtained from pressure transducers, accelerometers, and strain gauges. Measurements of pressure are most direct but are always limited, and often not possible when the necessary penetration of the enclosure to install instruments is not allowed. Hence the unsteady internal pressure field is often inferred from data taken with accelerometers and strain gauges. In any case, because it is the fundamental variable of the motions, the pressure will serve here as the focus of our discussion.

Figure 1.29 shows examples of pressure records from measurements taken with three different solid propellant rockets. They show many of the features commonly observed for combustion instabilities. The transient records 1.29 (a) and (b) exhibit the exponential growth characteristic of a linear instability. That behavior is most clearly exposed by plotting the logarithm of the peak values versus time, giving a straight line because $p \sim e^{\alpha t}$ means $\log p \sim \alpha t$. The result is of course not precise because the oscillation is not a pure sinusoidal motion and indeed, the case 1.29(b) shows development of steep fronts, often preceding evolution into approximately triangular waveforms accompanied by an increase in the average burning rate and chamber pressure. In any case, the transient growth usually slows and the oscillation becomes a limit cycle possibly containing several frequencies. Figure 1.29(c) is an example showing behavior often observed for instabilities in solid rockets for which the best measurements of transients have been made.

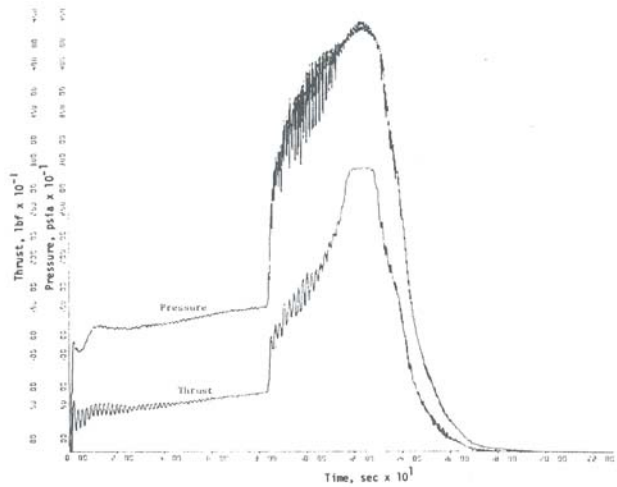
Figure 1.30 shows details of a test record taken during firing of a full-scale solid rocket. Development of steep waves and higher harmonics are clearly evident. Note also the rise of mean pressure accompanying the appearance of higher harmonics at time D. As shown also in Figures 1.29 (b) and (c), that is not an



(a)



(b)



(c)

FIGURE 1.29. Transient growths and "limit cycles" of combustion instabilities.

unusual occurrence. In the case giving the records shown in Figure 1.30, the instability was eliminated with a change of the internal configuration, consistent with the ideas pursued in this book.

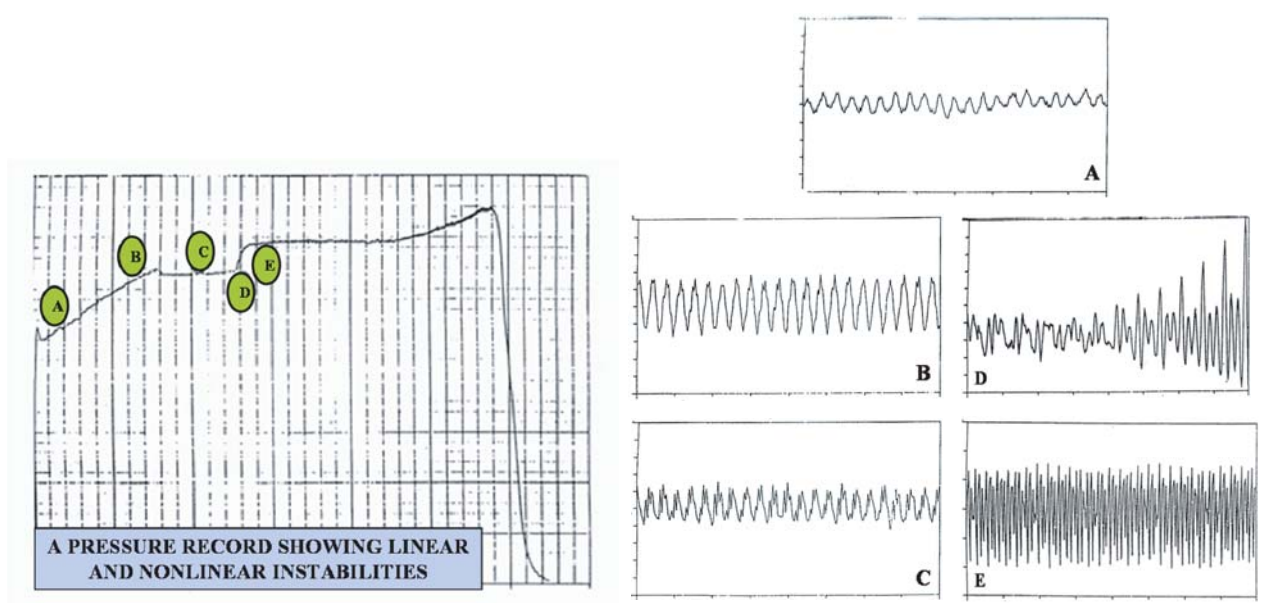


FIGURE 1.30. Growth of a nonlinear instability in a solid rocket showing the unsteady pressure. Note the increased spectral content of the signal as the amplitude increases.

A convenient and quickly informative method of displaying characteristics of an unstable firing is the "waterfall" plot, of which examples are shown in Figures 1.31 and 1.32. Spectra for a sequence of times are plotted on the same axes, the vertical axis serving as the measure of pressure as well as indicating the time intervals at which the spectra are taken. Figure 1.31 is the waterfall plot starting early in the test of a stable motor. A test having an instability, such as that shown in Figure 1.30 is reproduced in Figure 1.32.

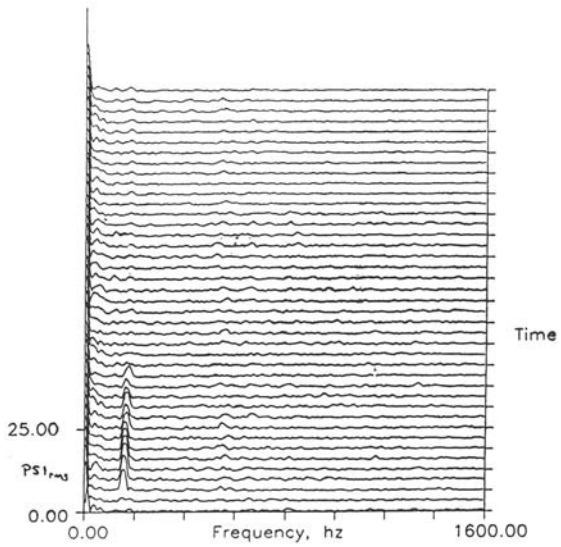


FIGURE 1.31. A waterfall plot taken during a stable firing (Brown 1995).

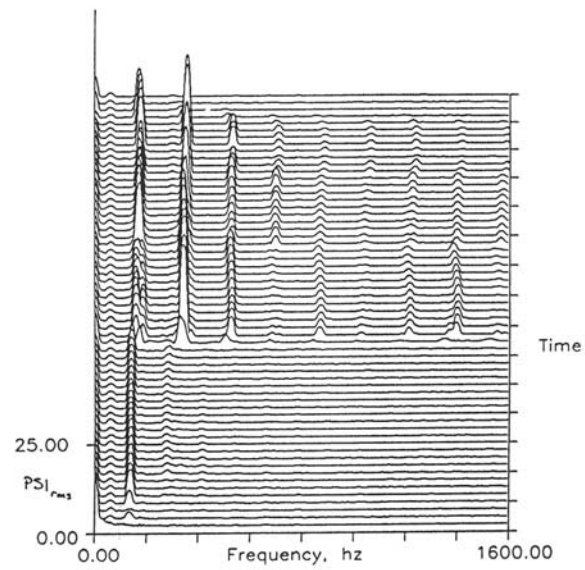


FIGURE 1.32. A waterfall plot showing an instability, having fundamental frequency < 200 Hz but with developing components at higher frequencies (Brown 1995).

The cases chosen here have been examples of fully developed instabilities. The well-defined peaks reflect the clear presence of several frequencies in the waves, the larger amplitudes occurring at the lower frequencies, as commonly happens. A substantial background of broad-band noise is of course always present due to turbulent fluctuations of the flow, noise emission by combustion processes, and possibly other unsteady motions such as flow separation. Some recent laboratory tests have shown that the level of noise depends on the presence and amplitude of combustion instabilities, but the cause is unknown and no such observations exist for full-scale combustors.

Much of this book is devoted to understanding the origins of the behavior illustrated by the examples in Figures 1.3, 1.6, 1.7, 1.29 and 1.30. The classical theory of acoustics has provided the basis for understanding combustion instabilities since early recognition that some unexpected observations could be traced to pressure oscillations. Many basic results of classical acoustics have been applied directly and with remarkable success to problems of instabilities. It is often taken for granted that well-known acoustics formulas should be applicable—their use can in fact be justified on fundamental grounds. However, in the first instance, it is surprising that they work so well, because the medium is far from the ideal uniform quiescent gas assumed in the classical acoustics of resonating chambers.

On the other hand, non-classical behavior is not difficult to find. It has been firmly established with tests using both liquid and solid rockets that instabilities involving "subcritical bifurcations" are common. That is, a combustor may be stable to small disturbances but may exhibit an instability if subjected to a

sufficiently large disturbance. An example of such behavior for a liquid rocket has been cited with Figures 1.6, 1.7 and 1.11. Figure 1.33 shows a typical result found for solid rockets. The subject of subcritical bifurcations is discussed in Section 7.11.

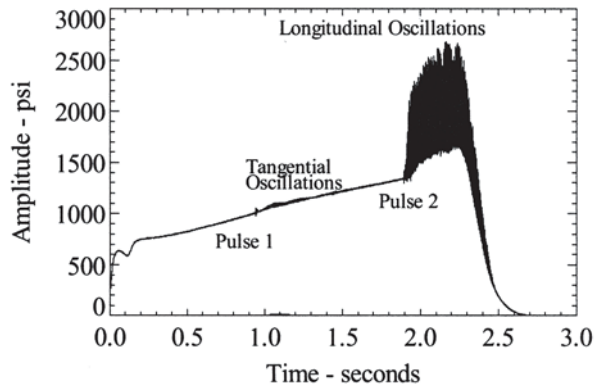


FIGURE 1.33. A severe pulsed instability in a solid rocket; an example of a subcritical bifurcation (Blomshield 2001).

A combustion chamber contains a non-uniform flow of chemically reacting species, often present in condensed as well as gaseous phases, exhausting through a nozzle that is choked in rockets, ramjets, and afterburners. Moreover, the flow is normally turbulent and may include regions of separation. Yet estimates of the frequencies of oscillations computed with acoustics formulas for the natural modes of a closed chamber containing a uniform gas at rest commonly lie within 10–15 percent or less of the frequencies observed for combustion instabilities, if the speed of sound is correctly chosen.

There are three main reasons that the classical view of acoustics is a good first approximation to wave propagation in combustion chamber: (1) the Mach number of the average flow is commonly small, so convective and refractive effects are small; (2) if the exhaust nozzle is choked, incident waves are efficiently reflected, so for small Mach numbers the exit plane appears to be nearly a rigid surface; and (3) in the limit of small amplitude disturbances, it is a fundamental result for compressible flows that any unsteady motion can be decomposed into three independent modes of propagation, of which one is acoustic (Chu and Kovazsnay 1956). The other two modes of motion are vortical disturbances, the dominant component of turbulence, and entropy (or temperature) waves. Hence even in the highly turbulent non-uniform flow usually present in a combustion chamber, acoustic waves behave in good first approximation according to their own simple classical laws. That conclusion has simplified enormously the task of gaining qualitative understanding of instabilities arising in full-scale systems as well as in laboratory devices.

Of course, it is precisely the departures from classical acoustics that define the class of problems we call combustion instabilities. In that sense, this book is concerned chiefly with perturbations of a very old problem, standing waves in an enclosure. That point of view has significant consequences; perhaps the most important is that many of the physical characteristics of combustion instabilities can be described and understood quite well in a familiar context. The remainder of this chapter is largely an elaboration of that conclusion.

The most obvious evidence that combustion instabilities are related to classical acoustic resonances is the common observation that frequencies measured in tests agree fairly well with those computed with classical formulas. Generally, the frequency f of a wave equals its speed of propagation, a, divided by the

wavelength, λ:

$$f = \frac{a}{\lambda} \tag{1.1}$$

On dimensional grounds, or by recalling classical results, we know that the wavelength of a resonance or normal mode of a chamber is proportional to a length, the unobstructed distance characterizing the particular mode in question. Thus the wavelengths of the organ-pipe modes are proportional to the length, L, of the pipe, those of modes of motion in transverse planes of a circular cylindrical chamber are proportional to the diameter, D, and so forth. Hence (1.1) implies

$$\begin{aligned} f &\sim \frac{a}{L} \quad \text{longitudinal modes} \\ f &\sim \frac{a}{D} \quad \text{transverse modes} \end{aligned} \tag{1.2)a,b}$$

There are two basic implications of the conclusion that the formulas (1.2)a,b, with suitable multiplying constants, seem to predict observed frequencies fairly well: evidently the geometry is a dominant influence on the special structure of the instabilities; and we can reasonably define some sort of average speed of sound in the chamber, based on an approximation to the temperature distribution. In practice, estimates of a use the classical formula $a = \sqrt{\gamma RT}$ with T the adiabatic flame temperature for the chemical system in question, and with the properties γ and R calculated according to the composition of the mixture in the chamber. Usually, mass-averaged values, accounting for condensed species, seem to be close to the truth. If large differences of temperature exist in the chamber, as in a flow containing flame fronts, nonuniformities in the speed of sound must be accounted for to obtain good estimates of the frequencies.

Even for more complicated geometries, notably those often used in solid rockets, when the simple formulas (1.2)a,b are not directly applicable, numerical calculations of the classical acoustic motions normally give good approximations to the natural frequencies and pressure distributions. Thus quite generally we can adopt the point of view that combustion instabilities are acoustical motions excited and sustained in the first instance by interactions with combustion processes. That the classical theory works so well for estimating frequencies and distributions of the unsteady motions means that computation of those quantities is not a serious test of a more comprehensive theory. What is required first of a theory of combustion instabilities is a basis for understanding how and why combustion instabilities differ from classical acoustics.

In particular, two global aspects of minor importance in much of classical acoustics, are fundamental to understanding combustion instabilities: transient characteristics and nonlinear behavior. Both are associated with the property that with respect to combustion instabilities, a combustion chamber appears to an observer to be a *self-excited system*: the oscillations appear without the action of externally imposed forces. Combustion processes are the sources of energy which ultimately appear as the thermal and mechanical energy of the fluid motions. If the processes tending to dissipate the energy of a fluctuation in the flow are weaker than those adding energy, then the disturbance is unstable.

1.5. Linear Behavior

When the amplitude of a disturbance is small, the rates of energy gains and losses are usually proportional to the energy itself which in turn is proportional to the square of the amplitude of the disturbance; the responsible processes are said to be linear because the governing differential equations are linear in the flow variables. An unstable disturbance then grows exponentially in time, without limit if all processes remain

linear. Exponential growth of the form $A_0 e^{\alpha t}$, where A_0 is the amplitude of the initial small disturbance, is characteristic of the initial stage of an instability in a self-excited system, sketched in Figure 1.34(a). In contrast, the initial transient in a linear system forced by an invariant external agent grows according to the form $1 - e^{-\beta t}$, shown in Figure 1.34(b). The curve $e^{\alpha t}$ is concave upward and evolves into a constant limiting value for a physical system only if nonlinear processes are active. However, the plot of $1 - e^{-\beta t}$ is concave downward and approaches a limiting value for a linear system because the driving agent supplies only finite power.

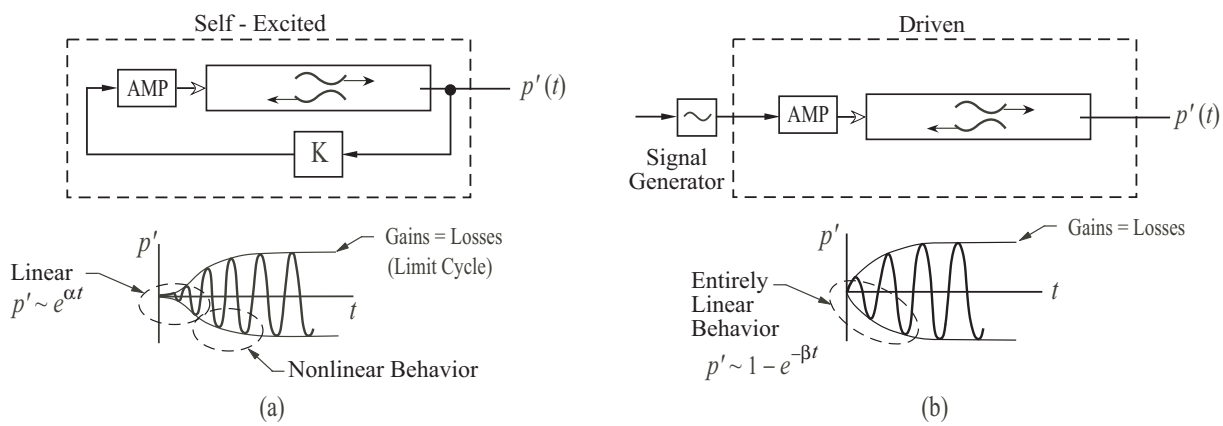


FIGURE 1.34. Transient behavior of (a) self-excited linearly unstable motions; (b) forced motions.

Data of the sort sketched in Figure 1.34 leave no doubt that the unstable motions in combustion chambers are self-excited, having the characteristics shown in Figure 1.34(a). The physical origin of this behavior is the dependence of the energy gains and losses on the motions themselves. For combustion instabilities, the 'system' is the dynamical system whose behavior is measured by the instrument sensing the pressure oscillations. Thus, in view of earlier remarks, the dynamical system is in some sense the system of acoustical motions in the chamber coupled to the mean flow and combustion processes (recall Figure 1.1).

It is a fundamental and extremely important conclusion that by far most combustion instabilities are motions of a self-excited dynamical system. Probably the most significant implication is that in order to understand fully the observed behavior, and how to affect and control it, one must understand the behavior of a nonlinear system. When the motion in a combustion chamber is unstable, except in unusual cases of growth to destruction, the amplitude typically settles down to a finite value: the system then executes a limiting motion, usually a periodic limit cycle. For practical applications, it is desirable to know how the amplitude of the limit cycle depends on the parameters characterizing the system. That information may serve as the basis for changing the characteristics to reduce the amplitude, the goal in practice being zero. In any case, good understanding of the properties of the limit cycle will also provide some appreciation for those variables which dominate the behavior and to which the motions may be most sensitive, a practical matter indeed.

Our global view, then, is that a combustion instability is an oscillatory motion of the gases in the chamber, which can in first approximation be synthesized of one or more modes related to classical acoustic modes. The mode having lowest frequency is a 'bulk' mode in which the pressure is nearly uniform in space but fluctuating in time. Because the pressure gradient is everywhere small, the velocity fluctuations are nearly zero. This mode corresponds to the vibration of a Helmholtz resonator obtained, for example, by blowing over the open end of a bottle. The cause in a combustion chamber may be the burning process itself, or it may be associated with oscillations in the supply of reactants, caused in turn by the variations of pressure in the chamber. In a liquid rocket, structural oscillations of the vehicle or the feed system may also participate, producing the POGO instability (Rubin 1966; Dordain, Lourme, and Estoueig 1974).

Structural vibrations of a solid rocket are not normally influential in that fashion, but an instability of the bulk mode (there is only one bulk mode for a given geometry) has often been a problem in motors designed for use in space vehicles. In those cases, the term L*-instability has been used because the stability of the mode is predominantly a function of the L* of the motor and the mean pressure (Sehgal and Strand 1964). The instability is associated with the time lag between fluctuations of the burning rate and of mass flux through the nozzles: That time lag is proportional to the residence time, and hence L*, for flow in the chamber. L*-instabilities occur in motors qualified for space flight because they arise in the lower ranges of pressure at which such rockets operate.

Whatever the system, most combustion instabilities involve excitation of the acoustic modes, of which there are an infinite number for any chamber. The values of the frequencies are functions primarily of the geometry and of the speed of sound, the simplest examples being the longitudinal and transverse modes of a circular cylinder, with frequencies behaving according to (1.2)a,b. Which modes are unstable depends on the balance of energy supplied by the exciting mechanisms and extracted by the dissipating processes. We consider here only linear behavior to illustrate the point.

In general the losses and gains of energy are strongly dependent on frequency. For example, the attenuation due to viscous effects typically increases with the square root of the frequency. Other sources of energy loss associated with interactions between the oscillations and the mean flow tend to be weaker functions of frequency. That is the case, for example, for reflections of waves by a choked exhaust nozzle. The gains of energy usually depend in a more complicated way on frequency.

The sources of energy for combustion instabilities, i.e. the mechanisms responsible for their existence, present the most difficult problems in this field. For the present we confine our attention to qualitative features of energy exchange between combustion to unsteady motions. For example, the magnitude of the energy addition due to coupling between acoustic waves and combustion processes for a solid propellant normally rises from some relatively small quasi-steady value at low frequencies, passes through a broad peak, and then decreases to zero at high frequencies. Recent experimental results suggest that flames may exhibit similar behavior (for example, Pun 2001). Energy is transferred to a pressure oscillation having a particular frequency at a rate proportional to the part of the coupling that is in phase with the pressure at that frequency.[5] Figure 1.35 is a schematic illustration of this sort of behavior.

In Figure 1.35, the gains exceed the losses in the frequency range $f_1 < f < f_2$. Modes having frequencies in that range will therefore be linearly unstable. An important characteristic, typical of combustion chambers generally, is that in the lower ranges of frequency, from zero to somewhat above the maximum frequency of instability, the net energy transfer is a small difference between relatively larger gains and losses. That implies the difficulty, confirmed by many years' experience, of determining the net energy flow accurately. Unavoidable uncertainties in the gains and losses themselves become much more significant when their difference is formed. That is the main reason for the statement made earlier that analysis of combustion instabilities has been useful in practice chiefly for predicting and understanding trends of behavior rather than accurate calculations of the conditions under which a given system is unstable. The ultimate source of all of these difficulties is the fact, cited in Section 1.1, that the motions in question consume and contain only small portions of the total energy available within the system. Hence in both laboratory tests and in operational systems one is confronted with the problem of determining the characteristics of essentially small disturbances imbedded in a complicated dynamic environment.

The best and most complete data illustrating the preceding remarks have been obtained with solid propellant rockets. There are several reasons for that circumstance. First, the ignition period — the time to cause all of the exposed propellant surface to begin burning — is relatively short and the average conditions in

[5] It is possible, due to the behavior of the phase, that in a range of high frequencies the combustion processes may in fact extract energy from the acoustic waves and hence contribute to the losses of energy.

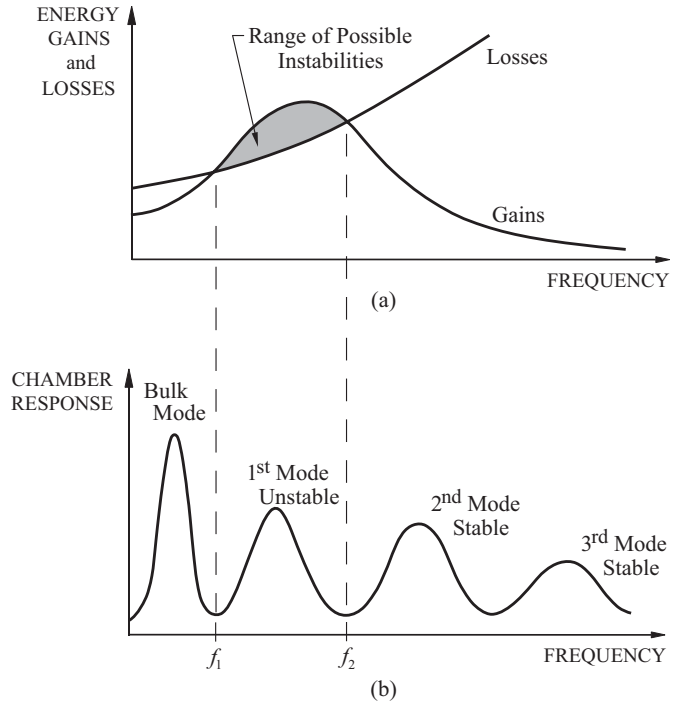


FIGURE 1.35. Qualitative dependence of (a) energy gains and losses; and (b) the frequency response of a combustor.

the chamber quickly reach their intended values. Unless oscillations are severely unstable, and grow rapidly during the ignition transient, there is a good opportunity to observe the exponential growth characteristic of a linear instability. The measurements shown in Figure 1.29 are good examples.

Secondly, it is probably true that more effort has been spent on refining the measurements and predictions of linear stability for solid rockets than for other systems because of the expense and difficulty of carrying out replicated tests. There is no practical routine way of interrupting and resuming firings, and it is the nature of the system that an individual motor can be fired only once. Particularly for large motors used in space launch vehicles, successive firings involve great expense. Development by empirical trial-and-error is costly and there is considerable motivation to work out methods of analysis and design applicable to individual tests.

In contrast, liquid-fueled systems can be fired repeatedly. Trial-and-error has long been a strategy for development of both liquid rockets and air-breathing systems. It seems that attention in that sort of work has generally been focused on modifications to reduce amplitudes (as in 'bombing' tests) rather than on determining the stability of small-amplitude motions. Very little data exists for values of growth constants, and most of those results have been obtained for model or sub-scale laboratory devices. There are examples of stability boundaries inferred from 'bombing' tests of the sort mentioned earlier and theoretical results exist, but there seem to be no investigations comparable to those carried out for solid rockets. A Standard Stability Prediction (SSP) program has been available for solid rockets for 25 years (Lovine *et al.* 1976, Nickerson *et al.* 1983) and is now the subject of a development program (French 2003); no such product in a general form exists for liquid rockets or for other propulsion systems.

We have already noted in Section 1.2 that much progress was achieved in analyzing and understanding combustion instabilities in solid rockets from the late 1960s into the 1980s when there was little work on

liquid rockets. During that period, computing resources, microprocessors, and therefore techniques of data acquisition and processing advanced enormously. Hence by comparison with the situation for solid rockets, the subject of combustion instabilities in liquid-fueled systems, especially liquid rockets, did not benefit as greatly from the general progress of supporting technologies. That situation is changing. Naturally, the problem of instabilities in gas turbines has only relatively recently been subject to widespread attention and therefore has not suffered the same historical deficiencies.

Finally, liquid or gaseous fueled systems are intrinsically more difficult to analyze and understand because of the more complicated chemical processes and coupling with the unsteady flow field. It is true that combustion of a heterogeneous solid propellant containing many ingredients, often including a metal, is very complicated indeed and far from completely understood in general. However, from the point of view of treating combustion instabilities, there is the great advantage that under most conditions, virtually all of the significant combustion processes are completed within a thin zone near the solid propellant itself. Coupling to the unsteady flow field may therefore be represented as a boundary condition. Combustion of liquid fuels is necessarily distributed throughout the volume of the chamber. Making accurate approximations to the spatial dependence is difficult, requiring quite careful treatment of many rate processes, including chemical kinetics and transfer of energy between liquid and gaseous phases. The elementary dynamics of the combustion processes are poorly understood relative to the situation for solid rockets.

1.5.1. Gains and Losses of Acoustic Energy; Linear Stability. It is a general result of the theory of linear systems that if a system is unstable, a small disturbance of an initial state will grow exponentially in time:

$$\text{amplitude of disturbance } \sim e^{\alpha_g t} \tag{1.3}$$

where $\alpha_g > 0$ is called the *growth constant.* If a disturbance is linearly stable, then its amplitude decays exponentially in time, being proportional to $e^{-\alpha_d t}$ and $\alpha_d > 0$ is the *decay constant.* The definition (1.3) implies that for a variable of the motion, say the pressure, having maximum amplitude $\hat{p}_0$ in one cycle of a linear oscillation:

$$p'(t) = \hat{p}_0 e^{\alpha_g (t - t_0)} \tag{1.4}$$

where $\hat{p}_0$ is the amplitude at time $t = t_0$. Then if p'_1, p'_2 are the peak amplitudes at time t_1, t_2,

$$\frac{\hat{p}_2}{\hat{p}_1} = \frac{p'(t = t_2)}{p'(t = t_1)} = \frac{e^{\alpha_g (t_2 - t_0)}}{e^{\alpha_g (t_1 - t_0)}} = e^{\alpha_g (t_2 - t_1)} \tag{1.5}$$

The logarithm of (1.5) is

$$\log \frac{\hat{p}_2}{\hat{p}_1} = \alpha_g (t_2 - t_1) \tag{1.6}$$

In practice, $t_2 - t_1$ is usually taken equal to the period τ, the time between successive positive (or negative) peaks. Then the logarithm of the ratio $\hat{p}_2/\hat{p}_1$ for a number of pairs of successive peaks is plotted versus the time t_1 or t_2 at which the first or second peak occurs. The line is straight, having slope α_g. Figure 1.36 shows a good example of both exponential growth and decay of pressure oscillations. The measurements were taken in a small laboratory device, a T-burner.

Whatever the system, the analytical treatment of linear stability is essentially the same. There is really only one problem to solve: find the growth and decay constants, and the frequencies of the modes. Determining the actual mode shapes is part of the general problem, but is often not essential for practical purposes. Typically, both the frequency and the mode shape for small-amplitude motions in a combustion chamber are so little different from their values computed classically as to be indistinguishable by measurement in operating combustors. By "classical" we mean here a computation according to the equations of classical acoustics for the geometry at hand, and with account taken of large gradients in the temperature, which affect the speed of sound. The presence of combustion processes and a mean flow field are not accounted for

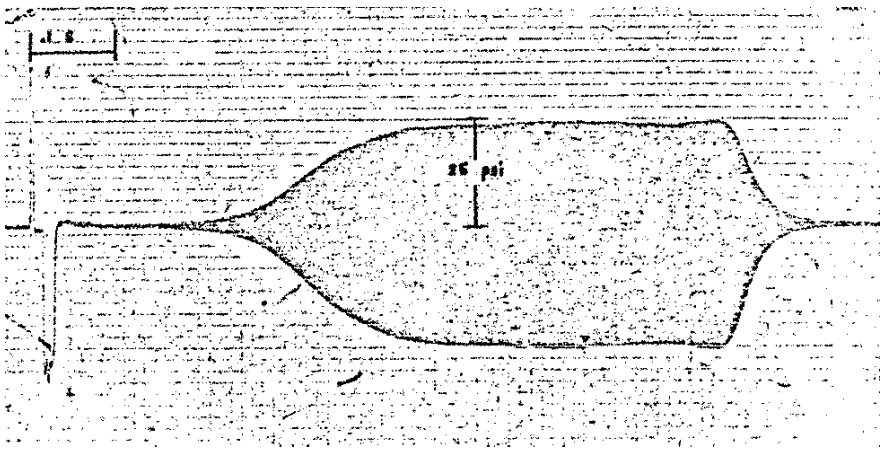


FIGURE 1.36. Exponential growth and development of a limit cycle out of a linearly unstable motion (Perry 1968).

explicitly, but it may be necessary, for satisfactory results, to include a good approximation to the boundary condition applied at the exhaust nozzle, particularly if the average Mach number is not small.

Hence the linear stability problem is really concerned with calculations of the growth and decay constants for the modes corresponding to the classical acoustic resonances. An arbitrary small amplitude motion can, in principle, be synthesized with the results, but that calculation is rarely required for practical applications. Results for the net growth or decay constant have been the central issue in both theoretical and practical work. In combustors, processes causing growth of disturbances and those causing decay act simultaneously. Hence an unstable disturbance is characterized by a net growth constant that can be written $\alpha = \alpha_g - \alpha_d$. Because the problem is linear, the growth constants can quite generally be expressed as a sum of the contributions due to processes accounted for in the formulation, as for example:

$$\alpha := \alpha_g - \alpha_d = (\alpha)_{\text{combustion}} + (\alpha)_{\text{nozzle}} + (\alpha)_{\text{mean flow}} + (\alpha)_{\text{condensed}} + (\alpha)_{\text{structure}} + \cdots \tag{1.7}$$

The labels refer to processes of interaction between the acoustic field and combustion, the nozzle, the mean flow, condensed species, the containing structure, ... Structural interactions comprise not only the vibrations mentioned earlier as a necessary part of the POGO instability, but also quite generally any motions of mechanical components, including propellant. For example, in large solid propellant rockets, motions of the viscoelastic material of the grain may be a significant source of energy losses through internal dissipation (McClure, Hart and Bird 1960a).

The stability boundary—the locus of parameters marking the boundary between unstable ($\alpha > 0$) and stable ($\alpha < 0$) oscillations—is defined by $\alpha = 0$ in (1.7). That statement is a formal statement of the physical condition that the energy gained per cycle should equal the energy lost per cycle:

$$\alpha_g = \alpha_d \tag{1.8}$$

Usually the main source of energy is combustion and in terms of the contributions shown in (1.7), this relation becomes

$$(\alpha)_{\text{combustion}} = -(\alpha)_{\text{nozzle}} - (\alpha)_{\text{mean flow}} - (\alpha)_{\text{condensed}} - (\alpha)_{\text{structure}} \tag{1.9}$$

There are situations in which the acoustic/mean flow interactions may provide a gain of energy. That is, energy is transferred from the average flow to the unsteady motions (as happens, for example, in wind instruments and sirens), but there is no need to consider the matter at this point.

As simple as it appears, equation (1.7) defining α, and its special form (1.8) defining the stability boundary, are basic and extremely important results. There is no evidence, for any propulsion system, contradicting the view that these results are correct representations of actual linear behavior. Difficulties in practice arise either because not all significant processes are accounted for, or, more commonly, insufficient information is available to assign accurately the values of the various individual growth or decay constants.

As examples, Figure 1.37 shows stability boundaries for longitudinal oscillations in a gas-fueled laboratory rocket motor (Crocco, Grey, and Harrje 1960) and Figure 1.38 shows the results of calculations for a large, solid propellant rocket (Beckstead 1974). Those examples illustrate the two uses mentioned above for the formula (1.9). In the case of the gas-fired rocket, the calculations contained two parameters not known from first principles, namely n and τ arising in the time-delay model of the interactions between combustion and the acoustic field. All other parameters defining the geometry and the average flow field were known. The purpose of the work was to compare the calculations with measurements of the stability boundary to infer values of n and τ.

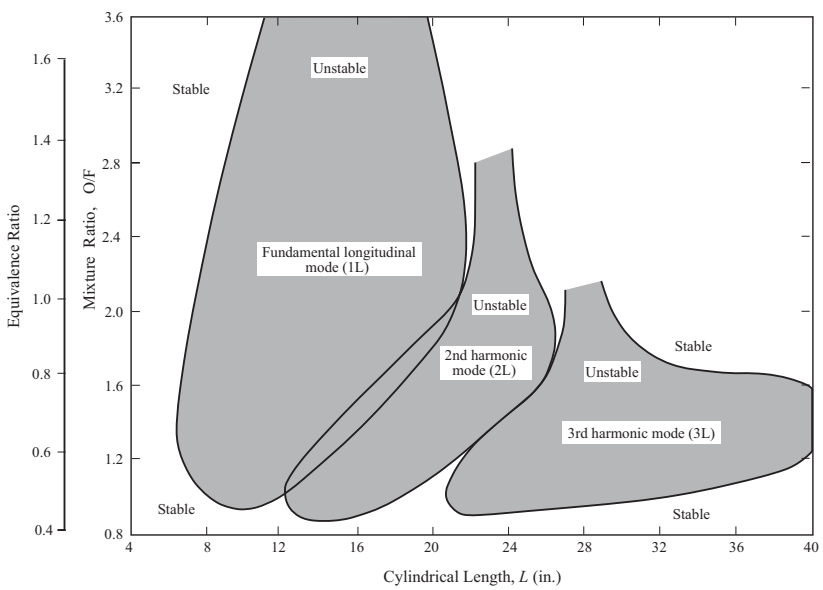


FIGURE 1.37. Stability boundaries for a laboratory gas-fueled rocket (Crocco, Grey, and Harrje 1960).

The purpose of the results reproduced in Figure 1.38 was to predict the stability of a full-scale motor prior to test firing. In that case, all of the parameters appearing in (1.7) must be known. Usually some of the information is available only from ancillary laboratory tests, notably those required to characterize the coupling between propellant combustion and the acoustic field.

Indeed, an important application of the formulas (1.11) and (1.12) is to do exactly that for the laboratory device called the "T-burner". It is not necessary to restrict attention to the stability boundary if good measurements of the growth constant can be made. Then if all the losses can be computed, one can find the value of the growth constant due to combustion (or, more generally all energy gains) as the difference

$$\alpha_{\text{combustion}} = \alpha - \alpha_{\text{losses}} \tag{1.10}$$

Results for $\alpha_{\text{combustion}}$ can either be adapted for use directly in computing the stability of a motor; or they can be interpreted with models of the combustion processes to obtain better understanding of unsteady combustion. That procedure has been used extensively to assess the combustion dynamics of solid propellants and to investigate trends of behavior with operating conditions and changes of composition.

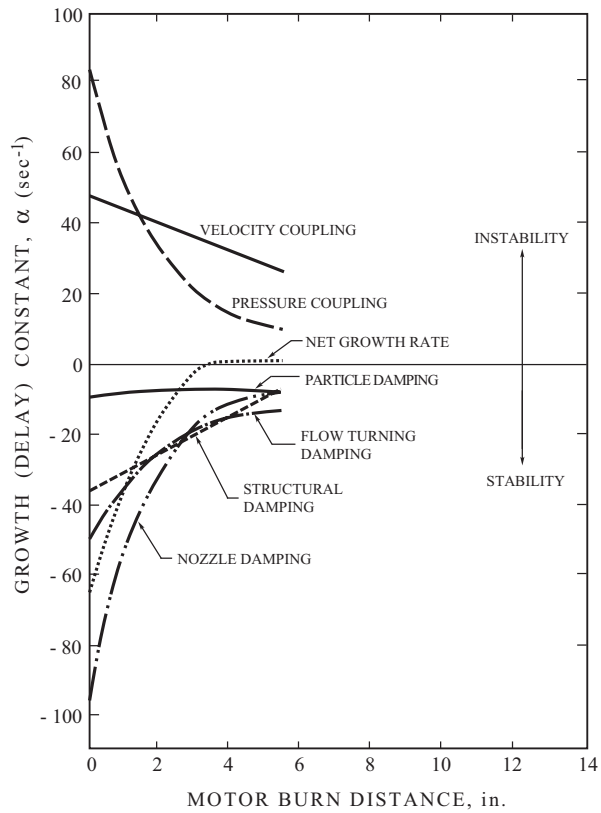


FIGURE 1.38. Predicted stability boundary for a large solid propellant rocket motor, and the separate contributions to α_d and α_g (Beckstead 1974).

The growth constant has a simple interpretation beyond that given by (1.6) as the slope of a semi-logarithmic plot of the peak amplitudes versus time: twice α is the fractional rate of change of time-averaged energy in the classical acoustic field. We will prove the result more rigorously in Chapter 5 but this interpretation is so central to all problems of linear stability that it is useful to have it in hand from the beginning. By the definition of α, both the pressure and velocity oscillations have the time dependence

$$p' \sim e^{\alpha t} \cos \omega t; \quad u' \sim e^{\alpha t} \sin \omega t$$

multiplied by their spatial distributions. The acoustic energy density is the sum of the local kinetic energy, proportional to u'^2, and potential energy, proportional to p'^2:

$$\text{K.E.} \sim e^{2\alpha t} \cos^2 \omega t; \quad \text{P.E.} \sim e^{2\alpha t} \sin^2 \omega t$$

If we assume that the period of oscillation, $\tau = 2\pi/\omega$, is much smaller then the decay rate, $1/\alpha$, then the values of these functions averaged over a cycle of the oscillation are proportional to $e^{2\alpha t}$. Hence the acoustic energy density is itself proportional to $e^{2\alpha t}$. Integrating over the total volume of the chamber we find that the total averaged energy $\langle \mathcal{E} \rangle$ in the acoustic field has the form

$$\langle \mathcal{E} \rangle = \langle \mathcal{E}_0 \rangle e^{2\alpha t} \tag{1.11}$$

where $\langle \mathcal{E}_0 \rangle$ is a constant depending on the average flow properties and the geometry. We then find directly from (1.11) the result claimed:

$$2\alpha = \frac{1}{\langle \mathcal{E} \rangle} \frac{d\langle \mathcal{E} \rangle}{dt} \tag{1.12}$$

Another elementary property worth noting is that $1/\alpha$ is the time required for the amplitude of oscillation to decay to $1/e$ of some chosen initial value. Also, the fractional change of the peak value in one cycle of oscillation ($t_2 - t_l = \tau = 2\pi/\omega$) is

$$|p_2'| - |p_1'| = \delta|p'|_m \sim e^{\alpha t_1} - e^{\alpha t_2} = e^{\alpha t_2}\left[e^{\alpha(t_1 - t_2)} - 1\right]$$

where $|\quad|_m$ denotes the magnitude of the peak amplitude. We assume as above that the fractional change in one period τ is small so

$$e^{\alpha(t_1 - t_2)} \approx 1 + \alpha(t_1 - t_2) = 1 + \alpha\tau$$

The amplitude itself is approximately proportional to $e^{\alpha t_2}$ or $e^{\alpha t_1}$ and we can write the fractional change as

$$\frac{\delta|p'|_m}{|p'|_m} \approx \alpha\tau = \frac{\alpha}{f} \tag{1.13}$$

where f is the frequency in cycles per second, $f = 1/\tau$. The dimensionless ratio f/α is a convenient measure of the growth or decay of an oscillation. According to the interpretation of $1/\alpha$ noted above, $(1/\alpha)/\tau = f/\alpha$ is the number of cycles required for the maximum amplitudes of oscillation to decay to $1/e$ or grow to e times an initial value.

The ratio α/f must be small for the view taken here to be valid. Intuitively, α must in some sense be proportional to the magnitude of the perturbations of the classical acoustics problem. We will find that the most important measure of the perturbations is a Mach number, $\bar{M}_r$, characterizing the mean flow; for many significant processes, α/f equals $\bar{M}_r$ times a constant of order unity. Roughly speaking, then, the measured value of α/f is an initial indication of the validity of the view that a combustion instability can be regarded as a motion existing because of relatively weak perturbations of classical acoustics.

1.6. Nonlinear Behavior

It is a fundamental and extremely important conclusion that combustion instabilities are motions of a self-excited nonlinear dynamical system. Probably the most significant implication is that in order to understand fully the observed behavior, and how to affect or control it, one must ultimately understand the behavior of a nonlinear system. When the motion in a combustion chamber is unstable, except in unusual cases of growth to destruction, the amplitude typically settles down to a finite value: the system then executes a limiting motion, usually a periodic limit cycle. For practical applications, it is desirable to know how the amplitude of the limit cycle depends on the parameters characterizing the system. That information may serve as the basis for changing the characteristics to reduce the amplitude, the goal in practice being zero. In any case, good understanding of the properties of the limit cycle will also provide some appreciation for those variables which determine the behavior, and to which the motions may be most sensitive, a practical matter indeed.

Rarely do the motions in a combustion chamber exhibit clear limit oscillations of the sort commonly encountered with simpler mechanical systems. A particularly good example of a limit cycle in a T-burner is shown in Figure 1.36. It appears that combustion devices are subject to influences, probably not easily identified, that prevent constant frequencies and amplitudes in the limit motions. The motions seem not to be limit cycles in the strict sense. However, experience gained in the past few years suggests that the deviations from the well-defined behavior of simpler systems are normally due to secondary influences. There are several possibilities, although not enough is known about the matter to make definite statements. Recent analysis (Burnley, 1996; Burnley and Culick, 1999) has demonstrated that noise, and interactions between random and acoustical motions can cause departures from purely periodic limit cycles appearing very similar to those found in pressure records for operating combustors (Figure 1.39). The random or stochastic motions are likely associated with flow separation, turbulence, and combustion noise.

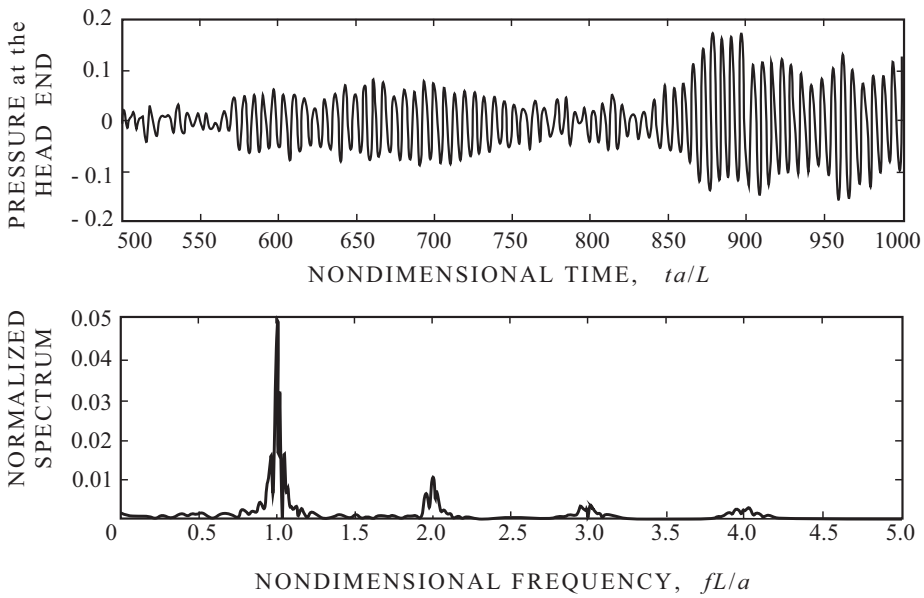


FIGURE 1.39. A computed limit cycle and its normalized spectrum executed by a single nonlinear acoustic mode in the presence of noise (Burnley and Culick 1996).

Probably other causes of departures from strictly periodic limit cycles are associated with the parameters characterizing steady operation of a combustor; and with 'noise' or random fluctuation of flow variables. As we have already emphasized, the unsteady motions require only a negligibly small part of the energy supplied by the combustion processes. Relatively minor variations in the combustion field, due, for example, to small fluctuations in the supplies of reactants, may alter the rates of energy transfer to instabilities and hence affect features of a limit cycle. Similarly, adjustments in the mean flow, notably the velocity field and surface heat transfer rates, will directly influence the unsteady field. Laboratory experiments clearly show such phenomena and considerable care is required to achieve reproducible results. In solid propellant rockets, the internal geometry necessarily changes during a firing. That happens on a time scale much longer than periods of unsteady motions, but one obvious result is the decrease of frequencies normally observed in tests. Because there is ample reason to believe that the phenomena just mentioned are not essential to the global nonlinear behavior of combustion instabilities, we ignore them in the following discussion.

1.6.1. Linear Behavior Interpreted as the Motion of a Simple Oscillator. Intuitively we may anticipate that nonlinear behavior may be regarded in first approximation as an extension of the view of linear behavior described in the preceding section, made more precise in the following way. Measurement of a transient pressure oscillation often gives results similar to those shown in Figure 1.34(a). The frequency in each case varies little, remaining close to a value computed classically for a natural resonance of the chamber, and the growth of the peak amplitude during the initial transient period is quite well approximated by the rule for a linear instability, $e^{\alpha t}$. Thus the behavior is scarcely distinguishable from that of a classical linear oscillator with damping, and having a single degree of freedom. The governing equation for the free motions of a simple mass (m)/ spring (k)/ dashpot (r) system is

$$m\frac{d^2x}{dt^2} + r\frac{dx}{dt} + kx = 0 \tag{1.14}$$

It is surely tempting to model a linear combustion instability by identifying the pressure fluctuation, p', with the displacement x of the mass. Then upon dividing (1.14) by m and tentatively replacing x by p', we have

$$\frac{d^2p'}{dt^2} + 2\alpha\frac{dp'}{dt} + \omega_0^2 p' = 0 \tag{1.15}$$

where $2\alpha = r/m$ and the undamped natural frequency is $\omega_0 = \sqrt{k/m}$. The familiar solution to (1.15) has the form of the record shown in the lower part of Figure 1.34(a), $p' = \hat{p}_0 e^{\alpha t} \cos \Omega t$ where $\Omega = \omega_0 \sqrt{1 - (\alpha/\omega_0)^2}$ and $\hat{\rho}_0$ is the value of p' at $t = 0$.

The preceding remarks suggest the course we should follow to investigate the linear behavior of combustion instabilities, and indeed is the motivation behind the general view described earlier. But this is purely descriptive heuristic reasoning. No basis is given for determining the quantities 'mass,' 'damping coefficient,' and 'spring constant' for the pressure oscillation. The procedure for doing so is developed in Chapter 4; the gist of the matter is the following, a brief description of the method used later to analyze combustion instabilities.

According to the theory of classical acoustics for a sound wave, we may identify both kinetic energy per unit mass, proportional to the square of the acoustic velocity u', and potential energy per unit mass, proportional to the square of the acoustic pressure p'. The acoustic energy per unit volume is

$$\frac{1}{2}\left(\bar{\rho} u'^2 + \frac{p'^2}{\bar{\rho}\bar{a}^2}\right) \tag{1.16}$$

where $\bar{\rho}$ and $\bar{a}$ are the average density and speed of sound. This expression corresponds to the formula for the energy of a simple oscillator,

$$\frac{1}{2}(m\dot{x}^2 + kx^2) \tag{1.17}$$

Now consider a stationary wave in a closed chamber. Both the velocity and pressure fluctuations have spatial distributions such that the boundary condition of no velocity normal to a rigid wall is satisfied. Hence the local pressure p' in equation (1.15) must depend on position as well as time. However, the frequency ω_0 depends on the geometry of the entire chamber and according to equation (1.12), we should be able to interpret 2α in equation (1.15) as the fractional rate of change of averaged energy in the entire volume. Therefore, we expect that the parameters m, k, and r implied by the definitions $\alpha = r/2m$ and $\omega_0 = k/m$ must be related to properties of the entire chamber. The approximate analysis used in most of this book is based partly on spatial averaging defined so that the properties ascribed to a particular mode are local values weighted by the spatial distribution of the mode in question, and averaged over the chamber volume.

Locally in the medium, the 'spring constant' is supplied by the compressibility of the gas, and the mass participating in the motion is proportional to the density of the undisturbed medium. When the procedure of spatial averaging is applied, both the compressibility and the density are weighted by the appropriate spatial structure of the acoustical motion. As a result, the damping constant and the natural frequency are expressed in terms of global quantities characterizing the fluctuating motion throughout the chamber. We will find rigorously that in the linear limit, an equation of the form (1.15) does apply, but instead of p' itself, the variable is $\eta_n(t)$, the time dependent amplitude of an acoustic mode represented by

$$p'_n = \bar{p}\eta_n(t)\psi_n(\vec{r}) \tag{1.18}$$

where $\bar{p}$ is the mean pressure and $\psi_n(\vec{r})$ is the spatial structure of the classical acoustic mode identified by the index $(\quad)_n$. Hence the typical equation of motion is

$$\frac{d^2\eta_n}{dt^2} + 2\alpha_n \frac{d\eta_n}{dt} + \omega_n^2 \eta_n = 0 \tag{1.19}$$

The constants α_n and ω_n contain the influences of all linear processes distinguishing the oscillation in a combustion chamber from the corresponding unperturbed classical motion governed by the equation

$$\frac{d^2\eta_n}{dt^2} + \omega_{n0}^2 \eta_n = 0 \tag{1.20}$$

if dissipation of energy is ignored. Because damping in a mechanical system causes a frequency shift, the actual frequency is not equal to the unperturbed value, ω_{n0}.

For technical reasons not apparent at this point, it is convenient to regard the linear perturbing process as a force $F_n(\eta_n, \dot{\eta}_n)$ is acting on the 'oscillator' and equation (1.19) is written

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = F_n^L(\eta_n, \dot{\eta}_n) \tag{1.21}$$

The superscript $(\quad)^L$ identifies the 'force' as linear, and for simplicity ω_{n0}^2 is written ω_n^2. We will consistently use the symbol ω_n for the unperturbed classical acoustic frequency. If there is no linear coupling between the modes (typically linear coupling is small), the force F_n^L consists of two terms, one representing the damping of the mode and one the frequency shift:

$$F_n^L = -\Delta\omega_n^2\eta_n + 2\alpha_n\dot{\eta}_n \tag{1.22}$$

Equations (1.21) and (1.22) produce (1.19) with ω_n^2 replaced by $\omega_n^2 + \Delta\omega_n^2$.

With the above reasoning we have heuristically constructed equation (1.21) as the fundamental equation for a linear combustion instability corresponding to a classical acoustic mode of the chamber. Its simplicity masks the fact that a great amount of effort is required to determine realistic functions $F_n^L(\eta_n, \dot{\eta}_n)$ applicable to the motions in a combustion chamber. The approximate analysis developed later provides a framework for accommodating all linear processes but does not contain explicit formulas for all of them. Most importantly, there are terms representing interactions between combustion processes and the unsteady motions, but their computation requires modeling the mechanisms that cause combustion instabilities. Some of the purely gasdynamical processes, arising with coupling between mean and fluctuating motions, are given explicitly.

According to classical acoustic theory, a closed chamber of gas at rest has an infinite number of normal or resonant modes. The spatial structures (mode shapes) and resonant frequencies are found as solutions to an eigenvalue problem. A general motion in the chamber, having any spatial structure, can then be represented as a linear superposition of the normal modes. The process of spatial averaging, leading to equation (1.20), amounts to representing any motion as an infinite collection of simple oscillators, one associated with each of the normal modes. That interpretation holds as well for equation (1.21) except that now each mode may suffer attenuation ($\alpha_n < 0$) or excitation ($\alpha_n > 0$). It is this point of view that allows natural extension of the analysis to nonlinear behavior.

1.6.2. Combustion Dynamics and Stability. Determining the linear stability of a system theoretically comes down to computing the value of the constant α, equation (1.21). With the model of an instability as a simple oscillator acted upon by a force dependent on the motion, the governing equation for the amplitude is (1.21). For simplicity, assume only one mode is active and that the driving force is entirely due to fluctuations of the rate of heat $\dot{Q}'$ provided to the flow. This type of motion is commonly called a 'thermo-acoustic instability.' In simplest form the equation for the amplitude $\eta_1(t) := \eta(t)$ is

$$\frac{d^2\eta}{dt^2} + \omega_1^2\eta = (\gamma - 1)\int \frac{\partial\dot{Q}'}{\partial t}\psi dV \tag{1.23}$$

where $\psi(\mathbf{r})$ is spatial distribution of the pressure for the mode, the 'mode shape', defined so the pressure fluctuation is $p' = \bar{p}\eta\psi(\mathbf{r})$. Derivation of equation (1.23) follows from the procedure worked out in Chapters 3, 4, and 6.

Suppose that the heat release rate is sensitive only to pressure and write its fluctuation as

$$\dot{Q}' = \frac{\dot{Q}'}{p'}p' = Rp' = \bar{p}R\eta\psi \tag{1.24}$$

where R is the response function, here having dimensions of inverse time,

$$[R] = \frac{[\text{Energy/Volume}]\frac{1}{t}}{[\text{Energy/Volume}]} = t^{-1}$$

Substitution of (1.24) in (1.23) and identification of α by comparison of the result with (1.19) leads to a formula for α:

$$\frac{d^2\eta}{dt^2} + \omega_1^2\eta = \left[(\gamma-1)\bar{p}\int R\psi^2 dV\right]\frac{d\eta}{dt} = 2\alpha\frac{d\eta}{dt} \tag{1.25}$$

and α is proportional to the response function,

$$\alpha = \frac{\gamma-1}{2}\bar{p}\int R\psi^2 dV \tag{1.26}$$

The equation governing the amplitude is

$$\frac{d^2\eta}{dt^2} - 2\alpha\frac{d\eta}{dt} + \omega_1^2 = 0 \tag{1.27}$$

with solution

$$\eta(t) = Ae^{\alpha t}\cos(\omega_t t + \phi) \tag{1.28}$$

where A and ϕ are constant and $\omega_t^2 = \omega_1^2 - \alpha^2$. Suppose R is a real constant, i.e. the fluctuation $\dot{Q}'$ of the heat release rate is in phase with the pressure fluctuation p'. If R, and hence α, is positive, the oscillation is driven by the response of the heat release to the pressure fluctuations.

This example is the simplest illustration of the direct connection between oscillations in a combustor and combustion dynamics represented here by a response function. The idea has enormously wide applications in all of the systems discussed in this book.

1.6.3. Nonlinear Behavior Interpreted as the Motion of a Nonlinear Oscillator. In view of the observation that measurements often show development of limit cycles like those shown in Figure 1.40, it is tempting simply to add a nonlinear term to the oscillator equation (1.21) and assume that a combustion instability involves only a single mode. Thus, for example, we could add to the right-hand side a force $F_n^{nL} = c_1\eta_n^2 + c_2\dot{\eta}_n^2 + c_3\eta_n\dot{\eta}_n + c_4|\eta_n| + \cdots$ where the constants $c_1, \ldots$ may be chosen by fitting the solution to data. Culick (1971) showed that quite good results could be obtained with this approach applied to limited data. Figure 1.40 shows one example. Of course this is a purely *ad hoc* approach and provides no means of computing the coefficients from first principles.

Following the early result shown in Figure 1.40, Jensen and Beckstead (1972) applied that procedure to extensive data taken in laboratory devices intended for measuring the characteristics of unsteady burning of solid propellants. The chief result was that the data could be matched equally well with rather broad ranges of the constants, and no particular kind of nonlinearity seemed to dominate the motions. Consequently, representation with a single mode was not successful. Even though analysis of pressure records for limit cycles often showed relatively small (it seemed) amounts of harmonics of the principle mode, it appeared necessary to account for two modes at least, with coupling due to nonlinear processes.

In other contexts, that conclusion is surely not surprising. The development of a small amplitude compressive disturbance into a shock wave is the oldest and most familiar example in gasdynamics. Steepening of a smooth wave arises primarily from two nonlinear influences: convection of the disturbance by its own motion, and dependence of the speed of sound on the local temperature, itself dependent on the wave motion. A good approximation to the phenomenon is obtained if viscous stresses and heat conduction are ignored. If the disturbance is regarded as a combination of various modes, the flow of energy from modes in the low frequency range to those having higher frequencies is favored by the nonlinear gasdynamic coupling. The rapid growth of the higher-frequency modes having shorter wavelengths produces the steepening, eventually

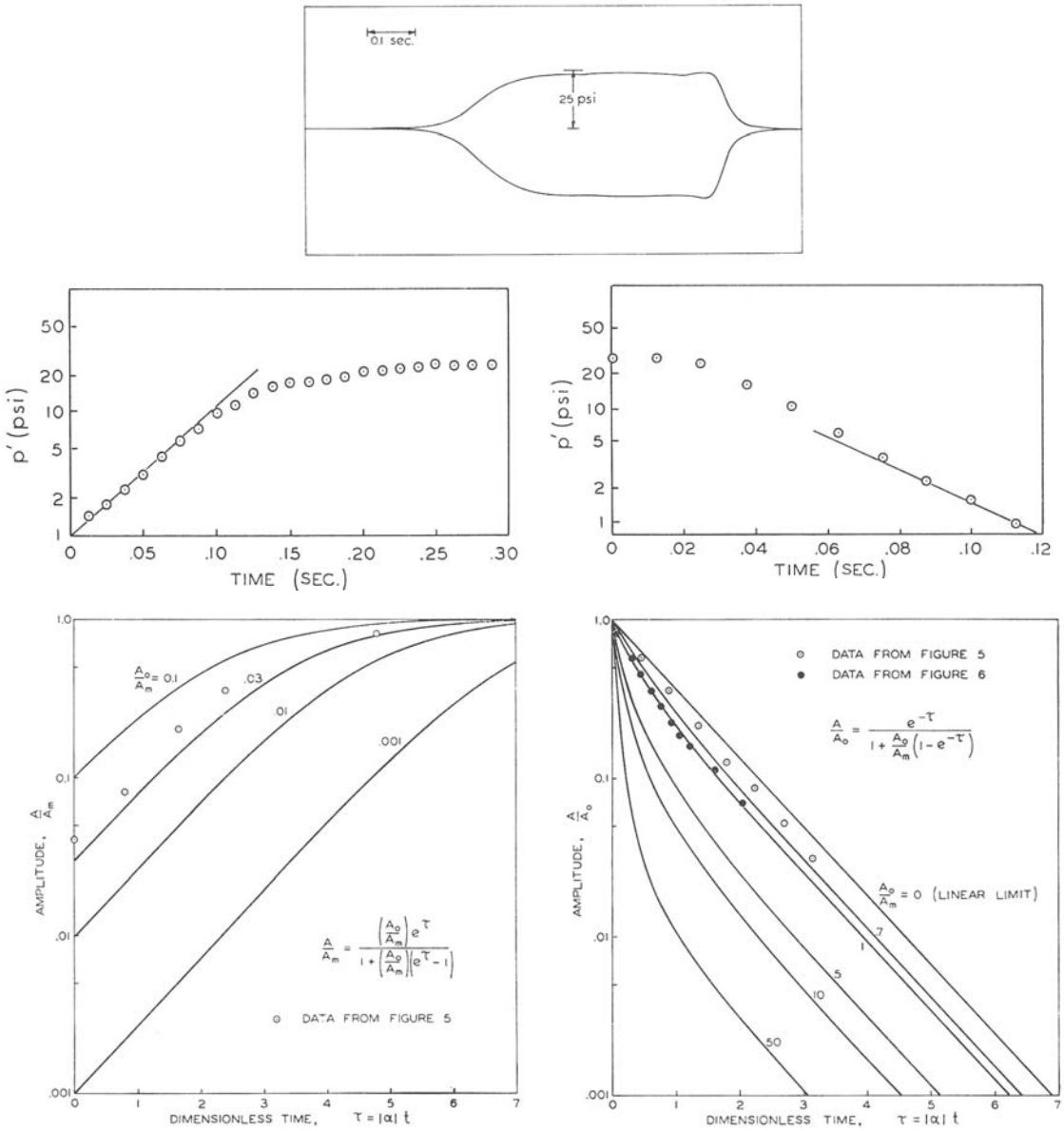


FIGURE 1.40. An example of fitting T-burner data with the model of a simple nonlinear oscillator (Culick 1971).

limited, in real flows, by the actions of various effects, mainly viscous. In a combustion chamber possible consequences of nonlinear combustion processes cannot be ignored. For example, there is much evidence accumulated in recent years that in some small gas-fueled combustors, the combustion processes may dominate the nonlinear behavior (see, for example, Dowling 1997).

In extreme cases of combustion instabilities, particularly in liquid and solid rockets, the approximately sinusoidal motions, substantially systems of stationary waves, may be absent or evolve into a different form. The motions then appear to be weak shock waves, or pulses having measurable width, propagating in the chamber. Instabilities of that type are commonly produced subsequent to excitation by finite pulses. Figure 1.41 shows examples observed in liquid rockets, typically involving motions mainly transverse to the axis.

They were discovered relatively early in the development of liquid rockets (e.g. Ross and Datner 1954) and were identified as 'spinning' transverse modes. Their presence is particularly harmful due to the greatly increased surface heat transfer rates causing unacceptable scouring of the chamber walls.

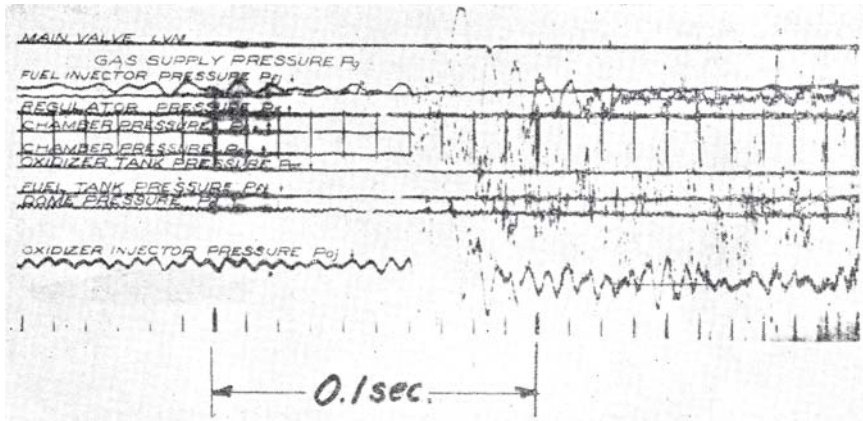


FIGURE 1.41. Steep-fronted waves observed in a liquid propellant rocket motor (Ross and Datner 1954).

The corresponding cases in solid rockets usually are longitudinal motions. They rarely occur in large motors and seem to have been first observed in pulse testing of laboratory motors (Dickenson and Jackson 1963; Brownlee 1960; Brownlee and Marble 1960). An example is reproduced in Figure 1.42 (Brownlee, 1964). Often this sort of instability is accompanied by a substantial increase of the mean pressure, seriously affecting the steady performance of the motors. The primary cause of the pressure rise is evidently the increased burn rate, although precisely why the rate increases is not well understood. More recently, these pulsed instabilities have been the subject of successful comparisons between laboratory test results and numerical simulations (Baum and Levine 1982; Baum, Levine, and Lovine 1988). Figure 1.43 shows an example of their results.

For combustion instabilities, the situation is very different from that for shock waves in a pure gas because the processes governing the transfer of energy from combustion to the gasdynamical motions cannot be ignored and in general depend strongly on frequency. Indeed, it may happen, as seems sometimes to be the case for combustion of solid propellants, that the coupling may cause attenuation of higher frequencies. For that reason, the tendency for steepening by the gasdynamics is partially compensated by the combustion processes, linear or nonlinear. As a result, in a chamber a limit cycle may be formed having very closely the spatial structure and frequency of the unstable mode (commonly, but not always, the fundamental mode) and relatively modest amounts of higher modes. It is that behavior that seems to be important in many combustion problems, explaining in part why the approach taken in the approximate analysis has enjoyed some success. Put another way, relatively small amounts of higher modes, presumed to arise from nonlinear behavior, may in fact represent important action by the nonlinear processes.

Naturally the preceding is a greatly simplified and incomplete description of the events actually taking place in a given combustion chamber. The essential conclusion that nonlinear gasdynamical processes are partly augmented and partly compensated by combustion processes seems to be an important aspect of all combustion instabilities. It appears that the idea was first explicitly recognized in work by Levine and Culick (1972, 1974), showing that realistic limit cycles could be formed with nonlinear gasdynamics and linear combustion processes. Perhaps the most important general implication of those works is that the nonlinear behavior familiar in flows of pure nonreacting gases is not a reliable guide to understanding the nonlinear behavior in combustion chambers.

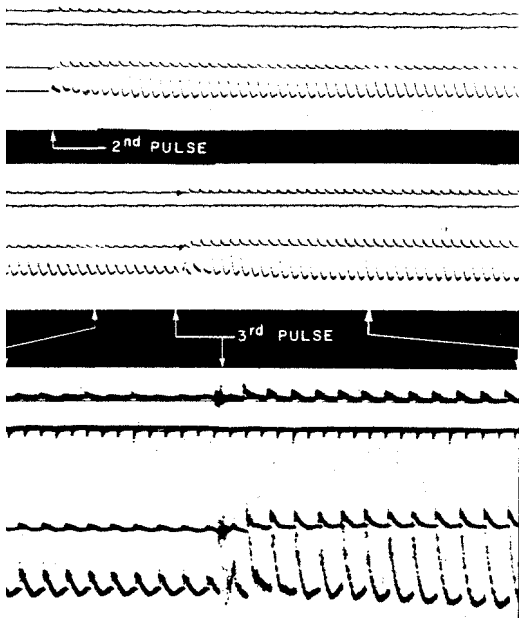


FIGURE 1.42. Steep-fronted waves observed in solid propellant rocket motors (Brownlee 1964).

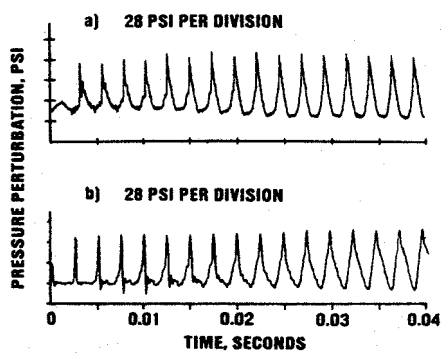


FIGURE 1.43. A comparison of observed and simulated steep-fronted waves in a solid propellant rocket motor. (a) observed; (b) numerical simulation (Baum and Levine 1982).

For nonlinear problems, the governing equations obtained after spatial averaging have the form

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = F_n^L(\eta_n, \dot{\eta}_n) + F_n^{NL}(\eta_i, \dot{\eta}_i) \tag{1.29}$$

where $F_n^{NL}(\eta_i, \dot{\eta}_i)$ is the nonlinear force depending on all amplitudes η_i, including η_n itself. Thus we may regard a combustion instability as the time-evolution of the motions of a collection of nonlinear oscillators, one associated with each of the classical acoustic modes for the chamber. In general the motions of the oscillators may be coupled by linear as well as nonlinear processes, although linear coupling seems rarely to be important. The analytical framework established by the dynamical system (1.29) will serve throughout this book as the primary means for analyzing, predicting, and interpreting combustion instabilities.

Representation of unsteady motions in a combustion by expansion in acoustic modes ('modal expansion') and application of spatial averaging was first accomplished by Culick (1961, 1963) using a Green's function. The work by Jensen and Beckstead cited above motivated extension to nonlinear behavior (Culick 1971 and

1975). Powell (1968) and Zinn and Powell (1970a, 1970b) first used an extension of Galerkin's method to treat nonlinear behavior in liquid rockets; the method was subsequently extended to solid rockets by Zinn and Lores (1972). In practice, application of a method based on modal expansion and spatial averaging is normally useful only if a small number of modes is required. Yet there are a large number of experimental results showing the presence of steep-fronted waves, often sufficiently steep to be interpreted as shock waves. Hence an analysis of the sort followed here would seem to be quite seriously limited unless one is prepared to accommodate a large number of modes. That is, one would expect that wave motions exhibiting rapid temporal changes and large spatial gradients must contain significant amounts of higher modes. However as discussion in this book will show, results have also established that due to fortunate phase relationships, a surprisingly small number of modes serves quite well even to represent many features of waves having steep fronts. The method gives quite a good approximation to both the limiting motions and the transient development of disturbances into weak shock waves.

1.7. Analysis and Numerical Simulations of Combustion Instabilities

In this book, the vehicle for unification is a theoretical/computational framework originating in the late 1960s and early 1970s with treatments of instabilities in liquid rockets (Culick 1961, 1963; Powell 1968; Zinn and Powell 1968; and Powell and Zinn 1971) and in solid rockets (Culick 1971, 1976). Those analyses differed from previous work mainly in their use of a form of spatial averaging, in some instances related to Galerkin's method, to replace the partial differential equations of conservation by a system of ordinary differential equations. The dependent variables are the time-dependent amplitudes of the acoustic modes used as the basis for series expansion of the unsteady pressure. It is the process of spatial averaging over the volume of the chamber that produces a formulation convenient for handling models of widely different geometries and physical processes. Consequently, in return for the approximate nature of the analysis (for example, the series must be truncated to a finite number of terms), one obtains both convenience and a certain generality of applications not normally possible when partial differential equations are used directly. In general form, this approach is applicable to all types of combustors. Different systems are distinguished by different geometries and the forms in which the reactants are supplied (liquid, solid, gas, slurry, ...). Those differences affect chiefly the modeling of the dominant physical processes.

Some analysis of combustion instabilities has customarily accompanied experimental work as an aid to interpreting observations. The paper by Grad (1949) treating instabilities in solid rockets is probably the first entirely theoretical work dealing with small amplitude acoustical motions in a mean flow field with combustion sources. During the 1950s and 1960s, many theoretical works were published on the subject of oscillations in solid rockets (e.g. Bird, McClure, and Hart 1963; Cheng 1954, 1962; Hart and McClure 1959, 1965; Cantrell and Hart 1964; Culick 1966) and in liquid rockets (e.g. Crocco 1952, 1956, 1965; Crocco and Cheng 1956; Reardon 1961; Culick 1961, 1963; Sirignano 1964; Sirignano and Crocco 1964; Zinn 1966, 1968, 1969; Mitchell, Crocco, and Sirignano 1969). It was during that period that the view of a combustion instability as a perturbation of classical acoustics was first extensively developed.

Most of the analyses cited in the previous paragraph were restricted to linear problems (those by Sirignano, Zinn and Mitchell are notable exceptions). Their chief purpose was to compute the stability of small amplitude motions. Indeed, since the earliest works on combustion instabilities, practical and theoretical considerations were directed mainly to the general problem of linear stability: the reasoning is that if the system is stable to small disturbances (e.g. associated with 'noise' always present in a combustion chamber) then undesirable instabilities cannot arise. There is a flaw in that reasoning: The processes in a combustion chamber are nonlinear, so a linearly stable system may in fact be unstable to sufficiently large disturbances. In any case, oscillations in combustors reach limiting amplitudes due to the action of nonlinear processes. Hence understanding nonlinear behavior is the necessary context in which one can determine what changes to the system may reduce the amplitudes. Ultimately, a complete theory, and therefore understanding, must

include nonlinear behavior, a subject covered at considerable length in this book, largely within the context cited in the first paragraph.

Recognition of the practical implications of the deficiencies of a view founded on linear behavior motivated the development of the technique of "bombing" liquid rocket chambers in the 1960s by NASA in its Apollo program (Harrje and Reardon 1972). The idea is to subject an operating combustion chamber to a succession of increasingly large disturbances (generated by small explosive charges) until sustained oscillations are produced. Then the size of the disturbance required to "trigger" the instability is evidently a measure of the relative stability of the chamber.[6] Another measure is the rate of decay of oscillations subsequent to a pulse injected into a linearly stable system. What constitutes the correct 'measure' of relative stability cannot of course be determined from experiments alone, but requires deeper understanding accessible only through theoretical work. This is part of the reason that the nonlinear analyses cited above were carried out; also, an extensive program of numerical calculations was supported (Priem and Hiedmann 1960; Priem and Guentert 1962; Priem and Rice 1968; and other works cited in the summary volume edited by Harrje and Reardon 1972). Owing to the limitations of computing resources at that time, those early numerical calculations involved solutions to quite restricted problems, commonly sectors or annular regions of chambers. It was not possible to compute what are now usually called 'numerical simulations.' Moreover, the results were often plagued—and were thus sometimes rendered invalid—by noise in the computations or difficulties with stability of the numerical techniques (for example, see Beltran, Wright, and Breen 1966).

While the intense activities on instabilities in liquid rockets nearly ceased in the early 1970s, work on numerical simulation of combustion instabilities in solid rockets began (Levine and Culick 1972, 1974; Kooker 1974; Baum and Levine 1982). In contrast to the case for liquid rockets, it is a good approximation to ignore chemical processes within the cavity of a solid rocket, an enormous simplification. Combustion occurs largely in a thin layer adjacent to the solid surface and its influences can be accommodated as boundary conditions. Consequently, with the growth of the capabilities of computers, it became possible to carry out more complete computations for the entire unsteady field in a motor. Also during this period appeared one of the earliest attempts to compare results of an approximate analysis with those obtained by numerical simulation for the 'same' problem (Culick and Levine 1974), a strategy which has since become generally accepted where it is applicable.

The main idea motivating that work was the following. At that time, the size and speed of available computers did not allow numerical simulations of three-dimensional problems, nor in fact even two-dimensional or axisymmetric cases. Moreover, no numerical calculations had been done of one-dimensional unsteady transient motions in a solid rocket, with realistic models of the combustion dynamics and partial damping. Approximate analysis of the sort mentioned above could be applied, in principle, to instabilities in arbitrary geometries, but owing to the approximations involved, there were no means of determining the accuracy of the results. Experimental data contain sufficiently large uncertainties that comparisons of analytical results with measurements cannot be used to assess accuracy of the analysis. Hence it appeared that the only way to assess the limitations of the approximate analysis must be based on comparison with numerical simulations. It was also important to confirm the validity of the approximate analysis because of its great value for doing theoretical work and for gaining general understanding of unsteady motions in combustion chambers.

That reasoning remains valid today. Despite the enormous advances in computing resources, it is true here as in many fields, that approximate analysis still occupies, and likely always will, a central position. A major reason is its great value in providing understanding. Numerical simulations advanced considerably during the 1980s and important work is currently in progress. Accomplishments for systems containing chemical processes, including combustion of liquid fuels, within the chambers far exceed those possible

[6] A quite different approach based on a statistical assessment of firings of many solid liquid propellant rockets of the same design was developed in Russia in the 1950s and 1960s. The method was unknown in the West, the first reports appearing in an international conference (Yang and Anderson, 1995).

twenty years ago (see, for example, the early papers by Liang, Fisher, and Chang 1986; Liou, Huang, and Hung 1988; Habiballah, Lourme, and Pit 1991; Kailasanath, Gardner, Boris, and Oran 1987a, b; Menon and Jou 1988).

Numerical simulations of flows in solid rockets began more than fifteen years ago to incorporate current ideas and results of turbulence modeling (Dunlop *et al.* 1986; Sabnis, Gibeling and McDonald 1985; Tseng and Yang 1991; Sabnis, Madabhushi, Gibeling and McDonald 1989). The results have compared quite favorably with cold flow experiments carried out using chambers with porous walls. There is no reason to doubt that eventually it will be possible to produce accurate computations of the steady turbulent flow fields in virtually any configuration expected in practical applications. A major step in that direction has been the recent emphasis on use of large eddy simulations (LES).

Remarkable success has also been achieved with computations of unsteady one-dimensional motions in straight cylindrical chambers (e.g. the early results reported by Baum and Levine 1982; Baum, Lovine, and Levine 1988; Tseng and Yang 1991). Particularly notable are the results obtained by Baum, Lovine, and Levine (1988) showing very good agreement with data for highly nonlinear unsteady motions induced in the laboratory by pulses. Although parameters in the representation of the unsteady combustion processes were adjusted as required to produce the good comparison, a minimal conclusion must be that the numerical methods were already quite satisfactory more than fifteen years ago.

Numerical simulation will always suffer some disadvantages already mentioned. In addition, because each simulation is only one case and the problems are nonlinear, it is difficult to generalize the results to gain fundamental understanding. However, the successes of this approach to investigating complicated reacting flows are growing rapidly and the methods are becoming increasingly important for both research and practical application. Historically, we have seen that the three aspects of the subject—experimental, analytical, and numerical simulations—began chronologically in that order. There seems to be no doubt that, as in many other fields of modern engineering, the three will coexist as more-or-less equal partners. We have therefore tried in this book to balance our discussion of methods and results of experiment, analysis, and numerical simulation with much less emphasis on the last. The integration of those activities forms a body of knowledge within which one may understand, interpret and predict physical behavior. For recent results applicable to combustion instabilities in solid propellant rockets, see the notes for a course given at the von Karman Institute (VKI) in 2001.

It is important to realize that experimental information about unsteady motions in combustion chambers is very limited. Commonly only measurements of pressure are available. Accelerometers and strain gauges mounted in a chamber may provide data from which some characteristics of the pressure field can be inferred. Quantitative surveys of the internal flow are virtually unavailable owing to the high temperatures, although optical methods are useful in laboratory work to give qualitative information and, occasionally, useful quantitative data.

As a practical matter we are therefore justified in assuming that only the pressure is available, at most as a function of time and position on the surface of the chamber. That restriction is a fundamental guide to the way in which the theory and methods of analysis for combustion instabilities are developed. Throughout this book we emphasize determining and understanding the unsteady pressure field. The approximate analysis constituting the framework in which we will discuss instabilities is based on the pressure as the primary flow variable. Very little attention will be given to methods of data analysis, an important activity directed chiefly to the problem of inferring quantitative properties of instabilities from pressure records.

1.8. A Précis of the Book

As an introduction to combustion instabilities, we intend to provide a broad foundation sufficiently deep to allow reading the contemporary literature without excessive difficulty, with the exception of modern work on numerical simulations. Many basic topics are simply not included, being assumed as part of the required general background, or outside the intended scope. The general character of the material covered is perhaps closer to applications than to basic research, but the roots in basic subjects are apparent throughout the text.

It is helpful to view the subject as a merging of three kinds of dynamics: chemical dynamics; combustion dynamics; and combustor dynamics; see the review by Culick (2000) which is concerned with solid rockets but some of the ideas apply to combustion systems generally. **Chemical dynamics** is concerned with behavior on a microscopic level and includes, among other topics, properties of elementary reactions (heats of reaction, activation energies, kinetic rate constants, ...); reaction paths and kinetic mechanisms; and products of reactions. **Combustion dynamics** implies the unsteady behavior of reacting systems exposed to variations in the environment, notably pressure, velocity and temperature. The relevant processes occur on a macroscopic scale—i.e. the medium may be regarded as a continuum for most purposes—but the scale is normally much smaller than that of a practical combustion chamber. Two large classes of problems, for example, comprise the dynamical responses of flames and of burning solid propellants.

Combustor dynamics is really the main subject of this book. Indeed, use of the term 'combustion instabilities' is dictated more by historical usage than for other reasons. These dynamics evolve on the scale of the combustion chambers in question. It is these dynamics that are observed in tests and that are responsible for the troublesome consequences in practical systems. However, **combustor** dynamics cannot be isolated from **combustion** dynamics, as explained in Section 1.1 and illustrated with Figure 1.1. The dynamics of a combustion chamber is the dynamics of two coupled systems: the medium supporting the motions and the combustion process. Put another way, we are concerned with a general problem of unsteady gasdynamics whose existence depends on the actions of one or more mechanisms almost always[7] arising from combustion dynamics.

Two characteristics distinguish different combustion chambers: geometry; and the kind of reactants (solids, liquids, gases, slurries, ...). In a formal sense, the details of the geometrical configuration are to a large extent secondary matters, particularly in the theory based on spatial averaging. There are significant quantitative differences among combustion systems arising from geometrical features, but one purpose of this book is to show that the characteristics of combustion instabilities common to all systems are far more significant and form much of the basis for understanding observed phenomena. Not surprisingly, then, the most difficult part of the subject is identifying, understanding and modeling the mechanisms. It is the mechanisms which most significantly differ among combustion systems. That is therefore the subject of Chapter 2. Then Chapter 3, and Annex A, are quite general, being concerned with the equations of motion and their forms most useful for the kinds of problems treated in the remainder of the book.

Much of Chapter 3 covers familiar ground, although the special use of the equations of motion for two-phase flow is perhaps not widely known. The chief result of that chapter is the wave equation for the pressure, constructed specially for treating nonlinear acoustic waves in combustion chambers. A significant distinction from most treatments of acoustics is that careful accounting for a general flow field is crucial matter for subsequent analysis of combustors; non-uniform average flow is an essential feature. In some way all analyses of combustion instabilities have taken advantage of the smallness of characteristic Mach numbers $\bar{M}_r$ and M'_r for the mean and unsteady flows. One strategy for constructing simplified forms of the conservation equations, based on the assumption that $\bar{M}_r$, M'_r are small, is explained in Chapter 3.

[7]The sole exceptions arise with mechanisms imbedded entirely within fluid mechanics.

A method of spatial averaging is then applied in Chapter 4, to replace the small set of partial differential equations by an infinite set of total differential equations. The procedure requires representation of the flow variables as a synthesis of an infinite set of basis functions. In this work the basis functions are classical acoustic modes for the chamber being studied, but the best choice is still a subject of research. Time-averaging, or the method of two-time scaling may be used to simplify the formal problem further by reducing the second-order equations, produced with spatial averaging, to first order equations. That step is significant for both theoretical and practical purposes.

The remainder of the book is patently parochial, being dominated by the character and results of work by the author, his students and his colleagues for many years. Chapter 5 is a quick review of those parts of acoustics required later. Practically all of the material can be found in several well-known texts, but some special requirements of the applications discussed here justify summarizing familiar material in accessible form. Only those topics are covered that arise in investigations of combustion instabilities.

Chapter 6 covers the basic linear theory of combustion instabilities proper. Probably two technical features most clearly distinguish the subject within the fields of combustion and fluid mechanics. First is the simultaneous presence of a mean flow and unsteady motion, both having relatively low Mach numbers. The second is the requirement that solutions be obtained within a bounded region of space for which the boundary conditions are mixed, ranging from perfectly reflecting to perfectly radiating. For most problems concerning stability of disturbances, experience has established that turbulence seems to play a very small part and can be ignored. Hence the various contributions to linear stability are connected in some way with the presence of an average flow and hence are measured by the characteristic Mach number $\bar{M}_r$ of the steady flow. The principal result is a formula for the wavenumbers of the perturbed acoustic modes, represented in three-dimensions, or within the one-dimensional approximation. For reasons given in Section 1.5, the emphasis in Chapter 6 is on solid rockets, but the general results apply to any type of system.

A broad spectrum of nonlinear behavior is covered in Chapter 7, beginning with examples of the special problems treated with the general analysis worked out in the preceding chapters. A continuation method is discussed in Section 7.7. Based on results obtained so far, this appears to be the most effective means of handling nonlinear combustion instabilities represented by the method of spatial averaging based on expansion in modes. Following a brief discussion of noise, some basic results obtained for pulsed instabilities (subcritical bifurcations) are covered.

The last two chapters are descriptive, discussions of practical methods for controlling combustion instabilities. Chapter 8 is a brief summary of ways to control instabilities passively. The motivations and the explanations for success of the methods rest on the fundamental behavior discussed in earlier chapters. However, in practice the actual problems are usually too involved to be treated analytically. Hence it is important to understand their basis and to obtain results for simple problems which can be solved.

Although it had been suggested more than 30 years earlier, active control became subject to vigorous research in the mid 1980s. But after rather intensive studies by many organizations, since roughly 2002 general interest in the subject has contracted, perhaps owing to the absence of successful applications. There have been many demonstrations that active control works to reduce the amplitudes of combustion instabilities, but no accompanying thorough explanations. Moreover, the power required has caused most cases to be impractical. Chapter 9 is a short coverage of some of the interesting results, including a practical application. Space and time restrictions have limited the present discussion which will be amplified in a later edition. It's a fascinating subject with wonderful possibilities, but achieving success for practical uses is a great deal more difficult and subtle than most people foresaw. The subject has a great future that must be built on basic understanding, which to a large extent is still absent.

It is not the intent of this book to provide only formulas and descriptions of problems. Our emphasis is on understanding the various kinds of combustion instabilities; and on developing and using one reasonably general method for analyzing and treating the phenomena in all types of propulsion systems. Experience over many years has shown that the approach followed here is useful in practice as well as for theoretical work. Familiarity with the basic principles, and with the procedure for their application, will give a sound context for understanding and attacking real problems.