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# Digital Antennas

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## *Abstract*

In this lecture, a brief history of antenna arrays will be given and trends in antenna beamforming will be presented. Conventional antenna arrays are built up of phase shifters, time delays, and power combiners, and the beamforming is done in the high frequency domain. Due to technology evolution ultra fast analog/digital converters are shifting the digitalization more and more towards the antenna. Field Programmable Gate Arrays (FPGA) have opened the way to beamforming in the digital domain. Analog components are replaced by mathematical operations. Therefore, nowadays antenna beamforming is performed in the digital domain. Even antenna signals in the GHz region and with several GHz bandwidth can be easily converted and processed.

The digitization promises more flexible radar systems, low power consumption, long term stability, multi-beam antennas, and adaptive beamforming. These days the main challenge in radar system design with a digital antenna array is to handle the huge data rate.

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<sup>0</sup>Weiß, M. (2009) Digital Antennas. In *Multistatic Surveillance and Reconnaissance: Sensor, Signals and Data Fusion* (pp. 5-1 — 5-29). Educational Notes RTO-EN-SET-133, Paper 5. Neuilly-sur-Seine, France: RTO. Available from: <http://www.rto.nato.int/abstracts.aps>.

# 1 Introduction

Antennas are widely used nowadays. One can find them in radar systems, wireless communication units, and satellite navigation systems. The employed antennas range from a single antenna with an unidirectional receive/transmit characteristic to complex 3D antenna arrays. The exponential growth in digital technology since the 80s, along with the corresponding decrease in cost, has a profound impact on the way how antenna arrays and therefore wireless communication and radar systems are designed. More and more functions that historically were implemented in analog hardware are now being performed digitally, resulting in increased performance and flexibility and reduced size and cost. Advances in analog-to-digital converter (ADC) and digital-to-analog converter (DAC) technologies are pushing the border between analog and digital processing closer and closer to the antenna. This paper deals with antennas, antenna arrays and the trends in digital beamforming that have become practical nowadays.

An antenna is a transducer designed to transmit or receive electromagnetic waves. In other words, antennas convert electromagnetic waves into electrical currents and vice versa. Antennas are used in systems such as radio and television broadcasting, point-to-point radio communication, wireless LAN, radar, and space exploration. Antennas usually work in air or outer space, but can also be operated under water or even through soil and rock at certain frequencies for short distances. A common antenna for wireless communication is a vertical rod a quarter of a wavelength long, as it is simple in construction, usually inexpensive, and both radiates in and receive from all horizontal directions (omnidirectional).

There are two fundamental types of antenna directional patterns, which, with reference to a specific three-dimensional (usually horizontal or vertical) plane, are either:

1. Omni-directional (radiates equally in all directions), such as a vertical rod, or
2. Directional (radiates more in one direction than in the other).

In colloquial usage *omni-directional* usually refers to all horizontal directions with reception above and below the antenna being reduced in favour of better reception (and thus range) near the horizon. A *directional* antenna usually refers to one focusing a narrow beam in a single specific direction such as a telescope or satellite dish, or, at least, focusing in a sector such as a 120° horizontal fan pattern in the case of a panel antenna at a mobile cell site.

All antennas radiate some energy in all directions in free space but careful construction results in substantial transmission of energy in a preferred direction and negligible energy

radiated in other directions. By adding additional elements (such as rods, loops or plates) and carefully arranging their length, spacing, and orientation, an antenna with desired directional properties can be created.

Typically, antennas are designed to operate in a relatively narrow frequency range. The design criteria for receiving and transmitting antennas differ slightly, but generally an antenna can receive and transmit equally well. This property is called reciprocity.

For some applications single element antennas are unable to meet the gain or radiation pattern requirements. Combining several single antenna elements in an array can be a possible solution. The next section introduces the basic concepts of antenna arrays.

## 2 Antenna array

Antenna arrays describe an antenna built up of a number of individual radiators. The radiated fields overlap, and forming a common antenna diagram through constructive interference. As a single radiator almost all antenna designs can be used. In a one-dimensional antenna array all individual elements are lined up mostly in a line and pointing in the same direction.

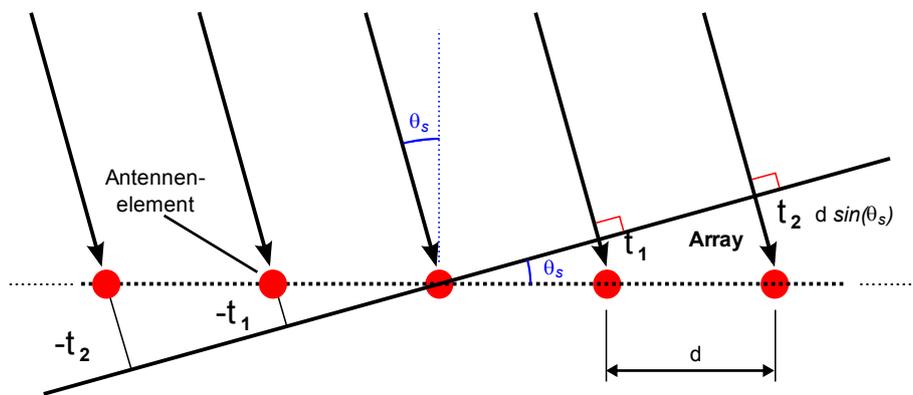


Figure 1: Linear antenna array

In Fig. 1 a linear equally spaced antenna array is shown, which consists of  $N$  individual elements. The distance between the adjacent antenna element is  $d$ . At each receiving element the following time delay  $\tau_n$  exists for an incident electromagnetic wave at an angle  $\theta_s$ :

$$\tau_n = \frac{n d}{c} \sin(\theta_s) \quad , \quad (1)$$

with  $c$  the speed of light,  $\theta_s$  the incidence angle and  $n = -\frac{N-1}{2}, \dots, \frac{N-1}{2}$  the numbering of each element relative to the antenna center.

When designing an antenna array it is fundamentally important to ensure that the element spacing is less than half a wavelength. If this constraint is not fulfilled the aliasing effect causes some sidelobes, which can become substantially larger in amplitude, and approaching the level of the main lobe. They are called grating lobes, and they are nearly identical copies of the main beam. Grating lobes are a special case of sidelobes. In such a case, the sidelobes should be considered all the lobes lying between the main lobe and the first grating lobe, or between grating lobes (Fig. 2). It is conceptually useful to distinguish between sidelobes and

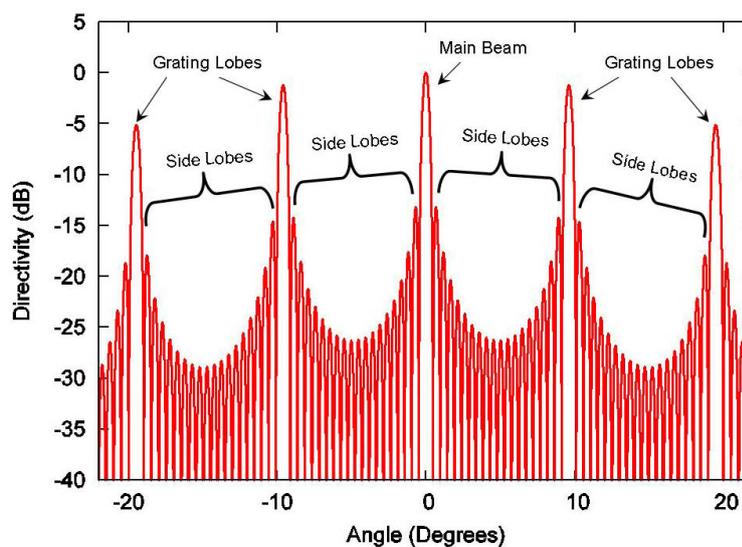


Figure 2: Typical antenna pattern with grating lobes

grating lobes because grating lobes have larger amplitudes than most, if not all, of the other side lobes.

For antennas used as receivers, sidelobes make the antenna more vulnerable to noise from nuisance signals coming from directions far away from the transmit source. For transmit antennas communicating classified information, sidelobes represent security vulnerability, as an unintended receiver may pick up the classified communication.

If this is taken into account, the optimum spacing between two adjacent elements is  $d \sim \lambda/2$  and the time delay for each element can be expressed by:

$$\tau_n = \frac{\lambda n}{2c} \sin(\theta_s) \quad . \quad (2)$$

Consider a plane wave with  $s_n(t) = s_0(t - \tau_n)$ , which impinges at an angle  $\theta_s$  on a linear antenna array with  $N$  receiving elements, each separated from its immediate neighbors by  $d$ .

If the received signal exhibits a small bandwidth  $B$  which meets the relation

$$B \ll \frac{c}{(N-1)d} \quad , \quad (3)$$

the signal can be considered as constant, while passing over all receiving elements. Accordingly, the individual time delays can be approximated by a complex phase factor:

$$s_n(t) = s_0(t) \cdot e^{-j \cdot 2\pi f_0 \cdot \tau_n} = s_0(t) \cdot e^{-j \cdot \varphi_n} \quad , \quad (4)$$

where  $f_0$  is the center frequency. Due to the equidistant spacing between the elements, the phase shift of each element depends only on the incident angle  $\theta_s$  and its location in the array:

$$\varphi_n = \frac{2\pi}{\lambda} d n \sin(\theta_s) \quad (5)$$

$$\varphi_n = \pi n \sin(\theta_s) \quad \text{with} \quad d = \lambda/2 \quad (6)$$

Taking into account this condition the following equation for the received signal for an uniform linear spaced array antenna can be written:

$$w_n = \frac{1}{N} e^{j n \pi \sin(\theta_s)} \quad . \quad (7)$$

To normalize the amplitude the factor  $\frac{1}{N}$  has been introduced. Summing up all signals from a one-dimensional antenna array looking into the direction of  $\theta_s$  with an incident signal  $S_0$  is:

$$S(\theta_s) = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} w_n(\theta_s) \cdot S_0 \quad . \quad (8)$$

### 3 Beamforming

Beamforming is an alternative name for spatial filtering where, with appropriate analog or digital signal processing, an array of antennas can be steered in a way to block the reception of radio signals coming from specified directions. While a filter in the time domain combines energy over time, the beamformer combines energy over its aperture (e.g. space), obtaining a certain antenna gain in a given direction while having attenuation in others. Beamforming has been used for many years in different radio applications such as communications, radio astronomy, wireless communication, surveillance, radar and, with different array sensors, in sonar and audio fields. It is a signal processing technique used in sensor arrays for directional signal transmission or reception. This spatial selectivity is achieved by using adaptive

or fixed receive/transmit beampatterns. Adaptive beamforming is used to detect and estimate the signal-of-interest at the output of a sensor array by means of data-adaptive spatial filtering and interference rejection. The improvement compared with an omnidirectional reception/transmission is known as the receive/transmit gain (or loss).

The traditional analog way to perform beamforming is very expensive, and it is sensitive to component tolerances and drifts, while modern technology offers high speed A/D converters and Digital Down Converters (DDCs), fundamental blocks for digital beamforming. In both analog and digital domains the most common methods used to create directional beams are the time delay (time shift) and phase shift ones.

The time delay approach allows to form and steer the beam by adding adjustable time delay steps that are independent from the operating frequency and bandwidth. Since it is difficult to generate time delays in both the analog and digital domains, they are used only when strictly requested as, for instance, with large arrays and/or when the bandwidth of the system is wide. In the case of phase shifting, a phase is introduced instead of applying true time delays for each receiver. It is simple to introduce such a compensation but unfortunately this works properly only with narrow band systems and/or small arrays. In practice the use of time or phase shift is determined by the loss of gain that can be accepted. The normalized gain depends on the bandwidth  $B$  and the delay  $\Delta\tau_{\Sigma} = N \Delta\tau$  (different time of arrival of the front wave at the antenna elements due to the physical dimension of the array) as reported in the following expression (normalized to 1):

$$G = \frac{\sin(\pi B \Delta\tau_{\Sigma})}{\pi B \Delta\tau_{\Sigma}} \quad (9)$$

In the receive beamformer the signal from each antenna may be amplified by a different weight  $w_n$ . Different weighting patterns (e.g. Dolph-Chebyshev) can be used to achieve the desired sensitivity patterns. A main lobe is produced together with nulls and sidelobes. As well as controlling the main lobe width (the beam) and the sidelobe levels, the position of a null can be controlled. This is useful to ignore noise or jammers in one particular direction, while listening for events in other directions. A similar result can be obtained on transmission.

Beamforming techniques can be broadly divided into two categories:

- conventional (fixed) beamformers
- adaptive beamformers

Conventional beamformers use a fixed set of weightings and time-delays (or phase-shifters) to combine the signals from the sensors in the array, primarily using only information about

the location of the sensors in space and the wave directions of interest. In contrast, adaptive beamforming techniques generally combine this information with properties of the signals actually received by the array, typically to improve rejection of unwanted signals from other directions. This process may be carried out in the time or frequency domains.

As the name indicates, an adaptive beamformer is able to adapt automatically its response to different situations. Some criterion has to be set up to allow the adaptation to proceed such as minimizing the total noise output. Because of the variation of noise with frequency, in wide band systems it may be desirable to carry out the process in the frequency domain.

### 3.1 Phased array

One of the biggest advantages of an electronically steered array is the capability of rapid and accurate beam scanning in microseconds which permits the radar to perform multiple functions either interlaced in time or simultaneously. Such radar is able to track a large number of targets and illuminate some of these targets with electromagnetic energy and guide missiles toward them. It can perform complete hemispherical search with automatic target selection and hand over to tracking. Very high transmit powers can be generated from multiplicity of high-power amplifiers connected to each antenna element. Electronically controlled phase array antennas can give radars the flexibility needed to perform all the various functions in a way suited to the specific task at hand. The antenna beams from phased arrays exhibits the flexibility to be designed as needed, for instance showing deep nulls in the direction of jammers while the main beam is nearly unbiased.

The relative amplitudes of and constructive and destructive interference effects among the signals radiated by the individual antennas determine the effective radiation pattern of the array. A phased array may be used to point a fixed radiation pattern, or to scan rapidly in azimuth or elevation.

In Fig. 3 a typical phased array configuration is depicted. It consists of  $N$  identical antennas, equally spaced at a distance  $d$ . On receive, if the plane wave is incident upon the array from a direction making an angle  $\theta_s$  with the array normal the  $n$ -th element will be of the form  $i_n = A_n e^{j k_0 n d \sin(\theta_s)}$ , with  $k_0 = 2\pi/\lambda$  the wavenumber at the center frequency of the signal. The summing network produces an output in the form:

$$S = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} A_n e^{j(\Phi_n + k_0 n d \sin(\theta_s))} \tag{10}$$

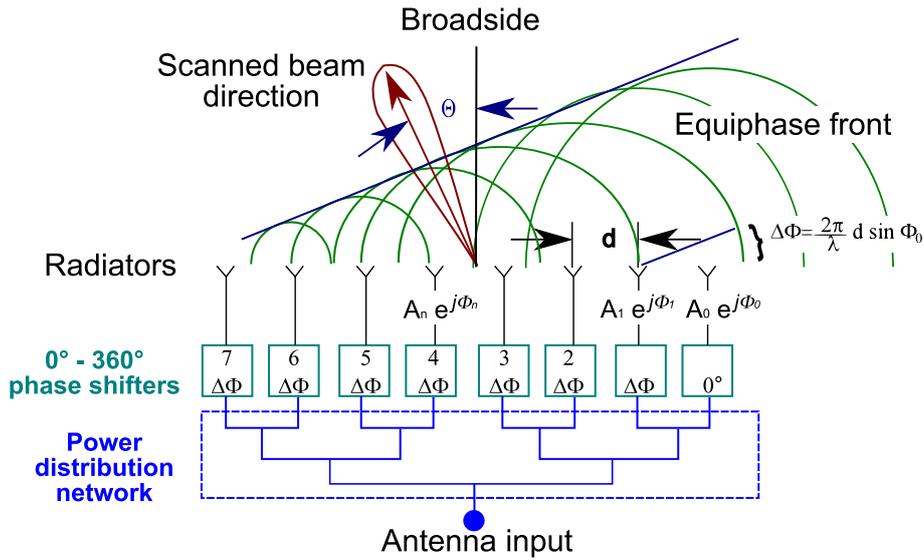


Figure 3: Beam steering concept using phase shifters at each antenna element

With  $\Psi_n$  the phase introduced by the  $n$ -th phase shifter. To combine the received signals from all antenna elements in phase to produce a maximum response in the scan direction of  $\theta_0$ , the  $\Phi_n$ 's must have the form

$$\Phi_n = -k_0 n d \sin(\theta_0) \tag{11}$$

This expression shows that the required phase taper across the array aperture is a linear taper (constant phase differential between adjacent antenna elements). Substituting Eq. 11 into Eq. 10, we get for the array

$$S = \sum_{n=-\frac{N-1}{2}}^{\frac{N-1}{2}} A_n e^{j k_0 n d (\sin(\theta_s) - \sin(\theta_0))} \tag{12}$$

Nowadays the phase-shifters are substituted by MEMS switches which toggle between different delay lines for each receiving element to overcome the bandwidth limit, as mentioned in Eq. 9.

## 4 Multi-beam Antennas

In radar and communications applications, it is necessary to scan a wide area. Array antennas, composed of  $N$  identical radiators, offer the capability to form many beams, up to  $N$ ,

in different directions from a single aperture by employing a passive beamforming network with an  $N \times N$  matrix. Increased use of multi-beam antennas is made possible through advances in design and manufacturing techniques, decreasing microwave material cost and improved design for beamforming networks. An RF signal entering anyone of the beamforming network ports excites all the antenna ports with the specified amplitude and phase distribution to produce the beam in a particular direction.

## 4.1 Butler-Matrix

The Butler-Matrix is one of the most popular multi-beam antenna approaches. It has been used extensively over the years in radar and electronic warfare (electronic support measures) and satellite systems. It is easy to implement and requires few components to build compared to other networks. The loss involved is very small, which comes from the insertion loss in hybrids, phase shifters and transmission lines. The Butler-Matrix produces  $N$  orthogonal beams overlapping at the 3.9 dB level having the full gain of an equally spaced  $N$ -element array.

The mechanism of a Butler-Matrix is understood by a discrete Fourier transform (DFT) of the Dirac delta function and translation rule ( $\mathcal{F}[f(t - t_1)] = \mathcal{F}[f(t)] \exp(j\omega t_1)$ ). The Fourier transform of the Dirac delta function  $\delta(t)$  is 1. When  $\delta(t)$  is translated in the time domain as  $\delta(t - t_1)$ , the taper in the phase is generated, and its Fourier transform becomes  $\mathcal{F}[\delta(t - t_1)] = \exp(j\omega t_1)$  due to translation rule. Analogous to the discrete Fourier transform, let us correspond the left and right ports on the Fig. 4 to the discrete time and frequency axis, respectively. The interpretation is now clear that the output ports are the discrete Fourier transform of the input ports. The Butler-Matrix is a microwave circuit which discrete Fourier transforms analog signals. The Fast Fourier transform (FFT) algorithm is used in the Butler-Matrix, and it is realized by using microwave circuits such as hybrids, phase shifters and cross junctions [1].

A Butler-Matrix requires  $\frac{N}{2} \log_2(N)$   $90^\circ$  hybrids interconnected by rows of  $\frac{N}{2} (\log_2(N) - 1)$  fixed phase shifters to form the beam pattern. Tapering will violate this requirement. For a cosine taper, for example, the cross-over level becomes approximately 9.5 dB for orthogonality. When a signal excites an input port of the Butler-Matrix, it produces different inter-element phase shifts between the output ports. The set of different inter-element phase shifts is given by

$$\Delta\theta = \pm(2k - 1) \frac{\pi}{2N}, \quad k \in [1, N] \quad (13)$$

where  $N$  is the number of ports of the matrix.

Consider the  $8 \times 8$  Butler-Matrix array shown in Fig 4. It consists of twelve  $90^\circ$  hybrids and eight fixed phase shifters that form a beamforming network. When one of the input ports is excited by an RF signal, all the output ports feeding the array elements are equally excited but with a progressive phase between them. This results in the radiation of the beam at a certain angle. For example if the 2R beam in Fig. 5 needs to be activated then the 2R input port needs to be activated. If multiple beams are required, two or more input ports need to be excited simultaneously. Fig. 5 shows the radiation of two beams 1R and 3L, which is achieved by simultaneous excitation of input ports 1R and 3L. Each beam can have a dedicated transmitter and/or receiver, or a single transmitter and/or receiver and the appropriate beam can be selected using an RF switch as mentioned earlier.

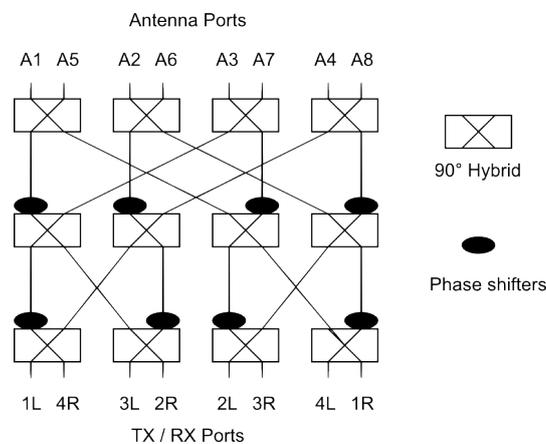


Figure 4:  $8 \times 8$  Butler-Matrix array

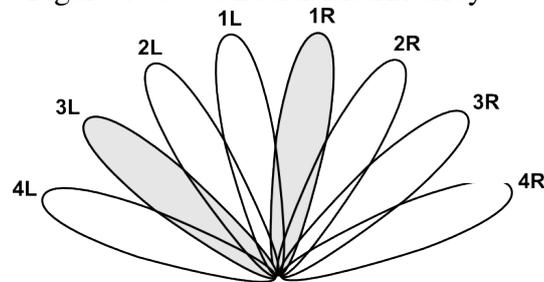


Figure 5: Radiation pattern of a  $8 \times 8$  Butler-Matrix array

The Butler-Matrix has been implemented using various techniques such as waveguide, microstrip, multilayer microstrip, suspended stripline, CPW. Microstrip technique is widely used to design Butler-Matrix due to its numerous advantages such as low profile, easy fabrication and low cost [4]. However a drawback of a Butler-Matrix is that beamwidth and beam angles tend to vary with frequency causing the beam to squint with frequency. To enhance the usable bandwidth of a Butler-Matrix emphasis has been put on the design of the hybrid.

For instance, 3-branch line hybrid was adopted to substitute a 2-branch line hybrid. An one-octave bandwidth hybrid has been achieved using multilayer coupled microstrip lines. Another method is that of using matching circuits which match the ports of hybrid. [5]

Moreover, as the matrices get bigger, more and more crossovers make interconnections complex. FGAN has realized a  $64 \times 64$  Butler-Matrix for OLPI (omnidirectional low probability of intercept radar). Fig. 6 shows the realized Matrix.

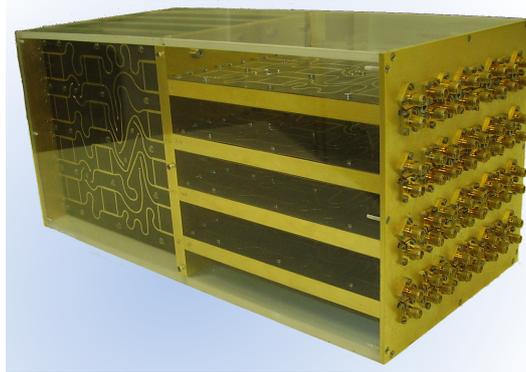


Figure 6: Realized  $64 \times 64$  Butler-Matrix at FGAN

## 4.2 Blass-Matrix

The Blass-Matrix is another microwave feeding network for antenna arrays with multi-beam capabilities. [6] The general scheme is depicted in Fig. 7.

The Blass-Matrix uses directional couplers and transmission lines to provide the necessary phase shift for the arrays in order to produce multiple beams. Fig. 7 shows an 8-element array fed by a Blass-Matrix. Each node represents a direction coupler to cross-connect the transmission lines. Port 0 provides equal delays to all elements and hence produce a broad side beam, whereas other ports provide progressive time delays between elements and hence produces beams at different angles. Therefore, when you send signal into the different inputs, you will get different steering angles.

The Blass-Matrix is a very flexible solution for multi-beam antennas, because it allows continuous scanning of the radiated beam direction; however, due to the presence of resistive terminations, it usually introduce more losses than other multi-beamforming solutions, such as the Butler-Matrix, although it is still possible to achieve high efficiency values (over 75%). Furthermore, while isolation for the first beam is ensured thanks to the high directivity of the

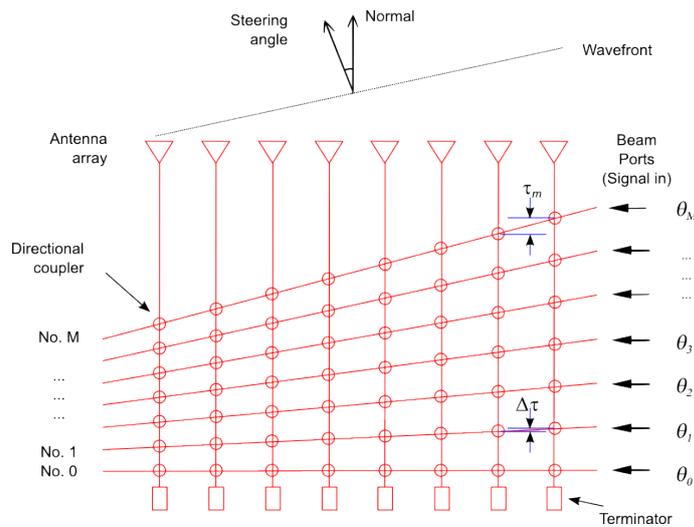


Figure 7: Schematic of a Blass-Matrix

coupler, the excitation of the other ports produces second order beams due to coupling with the first upper line.

### 4.3 Rotman Lens

Another possible solution to assemble a multi-beam antenna, especial for millimeter waves, is by a Rotman lens [7]. With the Butler and Blass-Matrix, the Rotman lens is an RF beam-former, which forms simultaneous multiple beams for an antenna array. It can be made using a pair of parallel plates with waveguides, probes or microstrip lines as input and output ports. Frequently Rotman lenses are designed for millimeter waves and there they offer inexpensive, rugged, reliable, and compact electronically-scanned antenna in comparison to mechanically steered scanning or phase shifters. Mechanically steered antennas are slow in response and suffer from reliability problems due to shock and vibration. Phase shifters are costly to fabricate and introduce considerable RF losses. By avoiding those drawbacks, the Rotman lens antenna could open new applications for millimeter wave radar. [8], [9] Production antennas employing a Rotman lens as a multi-beamforming network could be hot-pressed in plastic, which would then be coated with a conductor like gold. The antenna feed horns and switch array could be made the same way, allowing the antennas to be very low in cost.

Besides the low cost, compact size and ruggedness, the Rotman lens antenna also offers very low throughput loss and sidelobe emissions. In the prototype developed by Peterson and

Rausch, sidelobe power can be suppressed by a factor of one-thousand below the energy of the main beam. The power loss through the lens itself is less than 2 dB.

A schematic representation of a Rotman lens is shown in Fig. 8.

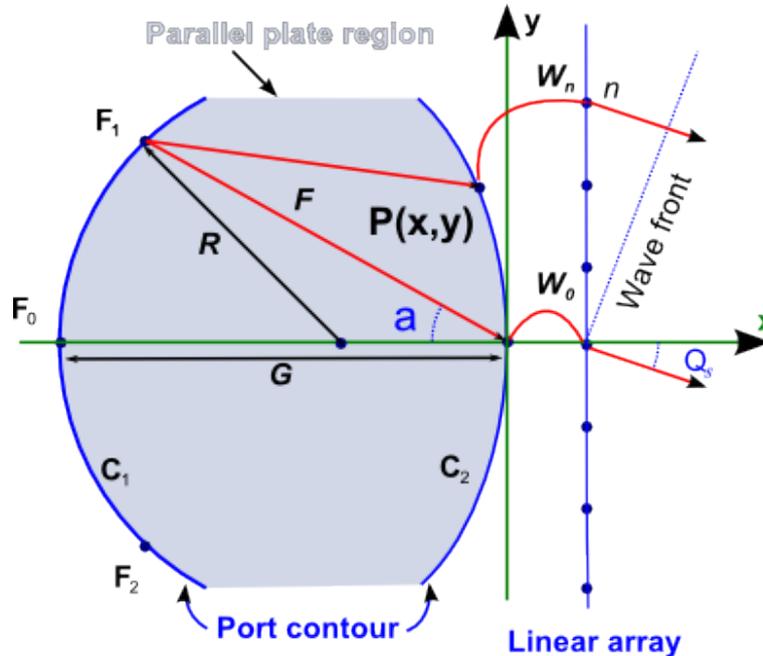


Figure 8: Rotman lens schematic layout

The parallel plate region is delimited by the port contour: the input ports are laying on a circular profile  $C_1$  while the output ports are laying on the profile  $C_2$  (also called array contour), defined by the two coordinates  $(x, y)'$ . The radiating elements are placed on a straight profile. Each output port on the profile  $C_2$  is connected to the corresponding  $n$ -th radiating element by a transmission line of length  $W_n$ . Two symmetrical off-axis focal points  $F_1$  and  $F_2$  and one on-axis focal point  $F_0$ , having coordinates  $(-F \cos(\alpha), F \sin(\alpha))$ ,  $(-F \cos(\alpha), -F \sin(\alpha))$  and  $(-G, 0)$ , are placed on the input profile. Let us denote with  $\Psi$  the desired steering angle of the radiation pattern when we feed from the focal point  $F_1$ , and with  $Y_n$  the vertical coordinate of the  $n$ -th radiating element.

Given the input parameters  $F$ ,  $G$ ,  $\alpha$ ,  $\Theta_s$  and  $Y_n$  the following equations allow to determinate the array profile  $(x, y)'$  and line lengths  $W_n$  in such a way that when a feed is placed at  $F_1$ ,  $F_2$  or  $G$  the radiation pattern is steered to  $-\varphi$ ,  $+\varphi$  and  $0^\circ$  with respect to the x-axis, respectively.

For a microstrip lens on a substrate with dielectric constant  $\epsilon_r$ , the following Rotman equa-

tions hold:

$$\sqrt{\varepsilon_r} (F_1 P) + \sqrt{\varepsilon_{eff}} W(N) + N \sin \Theta_s = \sqrt{\varepsilon_r} F + \sqrt{\varepsilon_{eff}} W(0) \quad (14)$$

$$\sqrt{\varepsilon_r} (F_2 P) + \sqrt{\varepsilon_{eff}} W(N) - N \sin \Theta_s = \sqrt{\varepsilon_r} F + \sqrt{\varepsilon_{eff}} W(0) \quad (15)$$

$$\sqrt{\varepsilon_r} (F_0 P) + \sqrt{\varepsilon_{eff}} W(N) = \sqrt{\varepsilon_r} G + \sqrt{\varepsilon_{eff}} W(0) \quad (16)$$

$$(F_1 P)^2 = (X + F \cos \alpha)^2 + (Y - F \sin \alpha)^2 \quad (17)$$

$$(F_2 P)^2 = (X + F \cos \alpha)^2 + (Y + F \sin \alpha)^2 \quad (18)$$

$$(F_0 P)^2 = (X + G)^2 + Y^2 \quad (19)$$

When the lens is fed from a generic angular position  $\varphi$  the main lobe is tilted at the angle  $\beta$ , which is determined by the following formula:

$$\sin \beta = \sin \varphi \cdot \frac{\sin \Theta_s}{\sin \alpha} \quad (20)$$

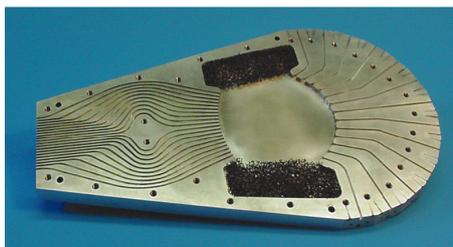


Figure 9: Manufactured Rotman lens for 94 GHz at FGAN (courtesy D. Nüßler)

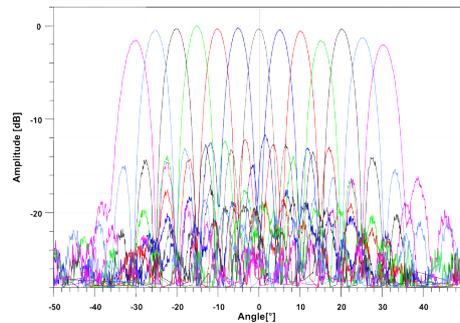


Figure 10: Measured beam of Rotman lens for 94 GHz at FGAN (courtesy D. Nüßler)

## 5 Digital beamforming

In the beginning of the 1980s, advances in digital circuitry technology made possible and feasible the idea of implementing the beamforming networks through digital signal processing.

**Digital beamforming (DBF)** offers advantages in terms of power consumption, flexibility, and accuracy. Furthermore advantages of DBF implementation can be seen in providing a high flexibility system using advanced adaptive algorithms, for instance providing null steering, accurate main-beam, superresolution, array element pattern correction, control of side-lobe levels, self calibration, multiple beam operation, and radar power and time management without changing the physical architecture of the phased array antenna. Every mode of operation of the digital beamformer is created and controlled by means of code written on a programmable device of the digital beamformer. In general, digital systems tend to consume less power in computation operations and have programmable interface adding versatility to the system. [3] Digital beamforming consists of the spatial filtering of a signal where the phase shifting, amplitude scaling, and adding are implemented digitally. The idea is to use a computational and programmable environment which processes a signal in the digital domain to control the progressive phase shift between each antenna element in the array.

Owing to major advances in analog-to-digital converter and digital component technology over the past years, digitization and signal processing are DBF is moving more and more towards the antenna elements, providing higher performance, and higher long-term stability at lower cost. The application of digital technology to IQ demodulation, which is just down-conversion of an IF signal to a complex baseband, has greatly improved the performance of coherent systems. One can group the digital downconverters (DDC) into two groups: a general form that is structurally parallel to traditional analog downconversion, and a restricted form, direct digital downconversion, which is more economical when it is applicable. Hence, less analog components are needed (see Fig. 11).

Antennas connected to ultra wideband digital receivers, which employ high speed analog-digital converters exhibiting sampling rates of 10 GHz and more, enable receiving simultaneously multiple narrowband signals. The sub-bands are digitally extracted with programmable band location and bandwidth control. Fig. 12 shows such a digital receiver capable of receiving three sub-bands concurrently.

To represent the digital downconverted signal, we simplify the mathematical representation of the signal  $g_n[m]$ , omitting the constant  $T_s$  in the signal  $x[mT_s]$  and using the variable  $\omega_{IF} = \omega_{IF} T_s$  to distinguish the cosine component in the digital signal representation from its analog representation. After making such simplifications, the digital signal observed in each DBF receiver channel  $n$  of the phased array antenna is

$$g_n[m] = g_n(t)|_{t=mT_s} = x[mT_s] \cos[\omega_{IF} m T_s - \Theta_n] \quad . \quad (21)$$

It is important to observe that the digital representation of the DBF receiver signal contains the phase delay  $\Theta_n$  associated with the time delay found in element  $n$  of the phased array

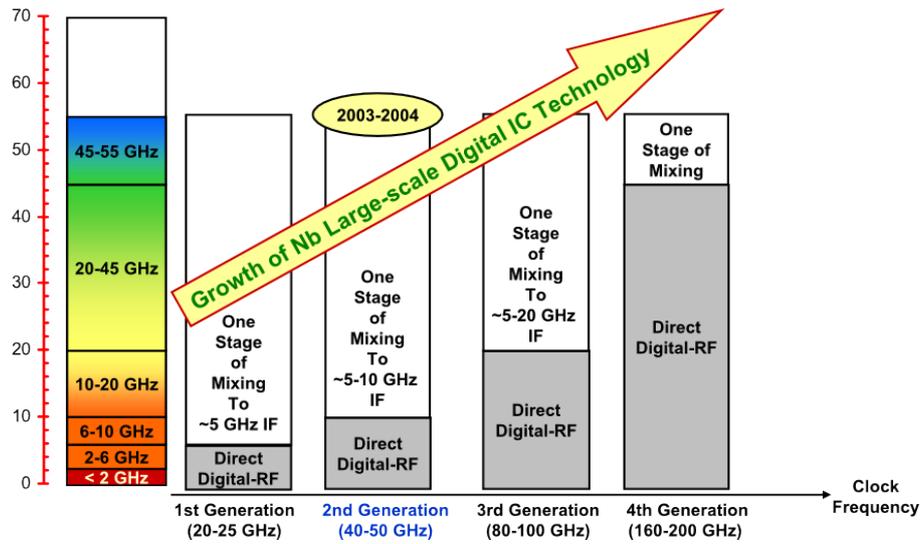


Figure 11: Technology growth in digital beamforming (Source: Hypres)

antenna.

After the antenna signal has been successfully sampled in the digital domain, the signal needs to be processed by the first stage of the DBF receiver, which is the Digital Down-Converter (DDC). The Digital Down-Conversion is performed by multiplying the digital signal with a sinusoidal signal and a 90° phase-shifted version of the sinusoidal signal, both generated by a local numerical controlled oscillator (NCO). Both mathematical operations can be represented in the following form:

$$\begin{aligned} i'_n[m] &= g_n[m] \cos[\omega_{NCO} m] \\ &= x[m] \cos[\omega_{IF} m - \theta_n] \cos[\omega_{NCO} m], \end{aligned} \quad (22)$$

$$\begin{aligned} q'_n[m] &= g_n[m] \sin[\omega_{NCO} m] \\ &= x[m] \cos[\omega_{IF} m - \theta_n] \sin[\omega_{NCO} m]. \end{aligned} \quad (23)$$

If the digital local oscillator frequency  $\omega_{NCO} = \omega_{IF}$ , the digital signals  $i'_n[m]$  and  $q'_n[m]$  for each DBF receiver channel can be represented in the following form:

$$i'_n[m] = \frac{x[m]}{2} (\cos[2\omega_{IF} m] + \cos[\theta_n]), \quad (24)$$

$$q'_n[m] = \frac{x[m]}{2} (\sin[2\omega_{IF} m] + \sin[\theta_n]). \quad (25)$$

The final step in the DDC stage of the DBF receiver is the filtering of the frequency component centered at the digital frequency  $2\omega_{IF}$  for both digital signals (image frequencies). If

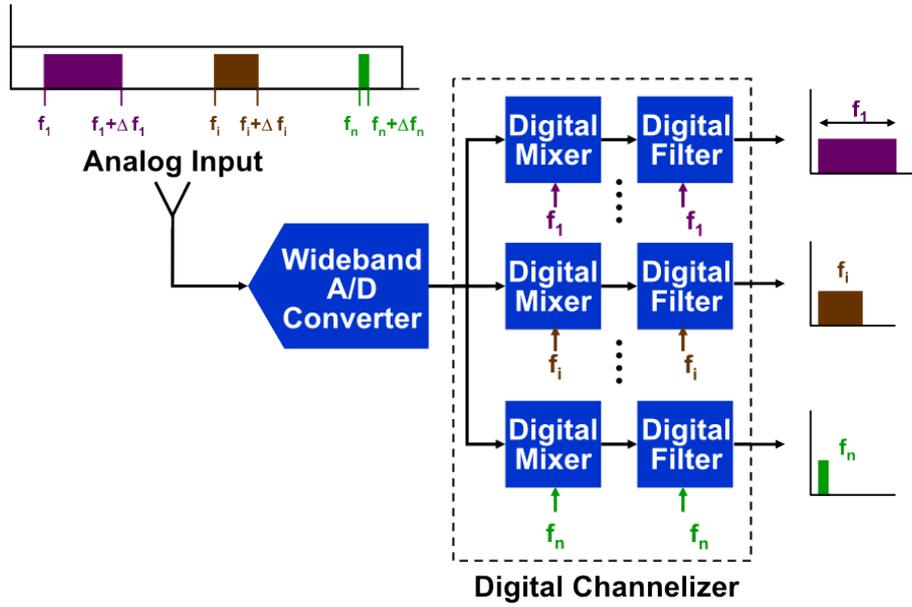


Figure 12: Multiband digital receiver

a lowpass filter with gain  $G = 2$  is used to process the signals  $i'_n[m]$  and  $q'_n[m]$ , the output signals found in each filter are:

$$i_n[m] = x[m] \cos[\theta_n] \quad (26)$$

$$q_n[m] = x[m] \sin[\theta_n] \quad (27)$$

It can be seen that the DDC stage of the DBF receiver transforms a digital bandpass signal with the time-delay  $\tau_n$  into two digital baseband signals where the phase information of the bandpass signal is represented in the amplitude of both baseband signals.

The previous transformation of the signal into its quadrature components is necessary in order to apply the next filtering phase as a double-input, double-output lowpass filter operation, which is equivalent to a single-input, single-output bandpass filter operation. Fig. 13 shows a block diagram of the RF modulator and the DDC stage of each antenna channel in the phased array antenna.

For beamforming, the complex baseband signals  $y_n[m]$  are multiplied by the complex weights  $w_n$  to apply the phase shift and amplitude scaling required for each antenna element.

$$w_n = a_n e^{j q_n} \quad (28)$$

$$w_n = a_n \cos(q_n) + j a_n \sin(q_n) \quad (29)$$

with  $w_n$  the complex weight for the  $n$ -th antenna element,  $a_n$  the relative amplitude of the weight, and  $q_n$  the phase shift of the weight. This stage allows to implement any weighting

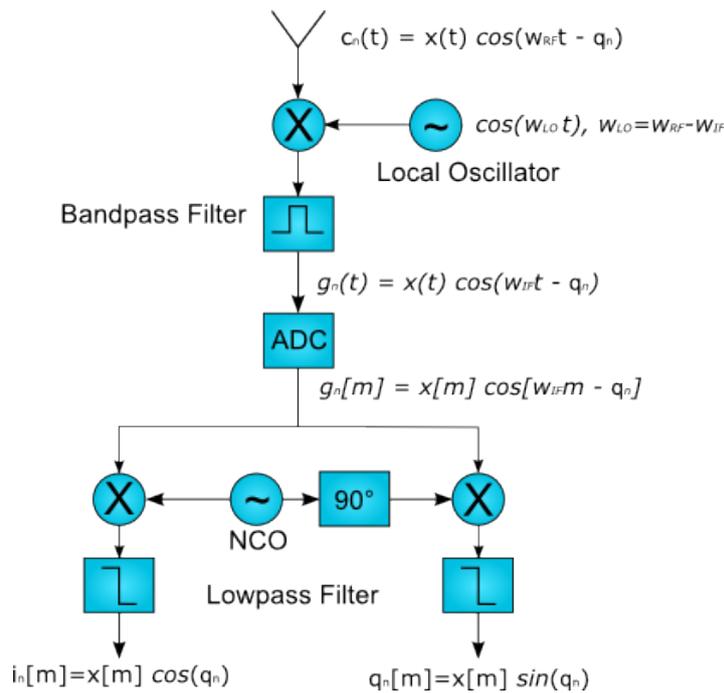


Figure 13: Block diagram (including equations) of RF modulator and DDC

like Hamming-Window, for instance. The last stage of the DBF receiver involves the addition of all the resulting signals  $y_n[m]$ :

$$y[m] = \frac{1}{N} \sum_{n=0}^{N-1} y_n[m] w_n \quad (30)$$

An amplitude scaling by a factor of  $N$  is needed to recover  $x[m]$  without gain.

Further implemented in the digital processing chain can be pulse compression, FFT, and IFFT for beamforming.

### 5.1 Adaptive Beamforming

The complex weights  $w_n$  for the antenna elements are carefully chosen to give the desired peaks and nulls in the radiation pattern of the antenna array. In a simple case, the weights may be chosen to give one central beam in some direction, as in a direction-finding application. The weights could then be slowly changed to steer the beam until maximum signal strength occurs and the direction to the signal source is found.

In beamforming for communications, the weights are chosen to give a radiation pattern that maximizes the quality of the received signal. Usually, a peak in the pattern is pointed to

the signal source and nulls are created in the directions of interfering sources and signal reflections.

Adaptive Beamforming is the process of altering the complex weights on-the-fly to maximize the quality of the communication channel. Here are some commonly used methods:

- **Minimum Mean-Square Error:** The shape of the desired received signal waveform is known by the receiver. Complex weights are adjusted to minimize the mean-square error between the beamformer output and the expected signal waveform.
- **Maximum Signal-to-Interference Ratio:** Where the receiver can estimate the strengths of the desired signal and of an interfering signal, weights are adjusted to maximize the ratio.
- **Minimum Variance:** When the signal shape and source direction are both known, choose the weights to minimize the noise on the beamformer output.

Often, constraints are placed on the adaptive beamformer so that the complex weights do not vary randomly in poor signal conditions. Some radio signals include training sequences so that an adaptive beamformer may quickly optimize its radiation pattern before the useful information is transmitted.

## 5.2 Multi-Beam Digital Beamforming

An important application of digital technology is the beamforming function in a phased array antenna system. In order to form a beam in a particular direction with an analog beamforming system, each element of the array needs to be followed by a time delay unit that delays the signal received at each element by the appropriate amount, such that when all of the outputs of the time delays are summed, they add up coherently to form a beam in the desired direction. If the system has a narrow bandwidth ( $B \ll 5\%$  of RF frequency) and the antenna beamwidth is not too narrow (so that the 3 dB beamwidth in degrees is greater than the percent bandwidth), the time delay can be approximated by phase shifters. Wide bandwidth systems require *true* time delays in order to form the beams and preserve the bandwidth. The receiver would follow the analog beamformer, which is capable of forming only a single beam at a time.

With a digital antenna array, where a receiver and ADC are behind every element, the time delay is implemented either as a digital phase shift or digital time delay, followed by a digital summer. This configuration allows beams to be formed in any direction, and multiple-beams

can be formed simultaneously, if desired, by using the same sample data and implementing different time delays to form the different beams.

Digital beamforming offers several advantages over analog beamforming. Radars are typically required to perform multiple functions, such as volume surveillance, target confirmation, tracking, etc. With only one beam at a time, there may not be enough time available to perform all of the required functions. With a digital antenna array, where a receiver and ADC are behind every element, it is possible to form multiple beams simultaneously. Hence, the volume surveillance function can be performed much more quickly, allowing more time to do other things. Of course, in order to form multiple simultaneous receive beams, the transmitted beam must be made broader to encompass the receive beams, which might require a more powerful transmitter or longer integration on receive to provide the same performance as a single-beam system. Another advantage has to do with dynamic range. In an analog beamforming system, there is only one receiver and ADC, and the dynamic range performance is limited to the capability of a single channel. In a digital beamforming system, there are multiple receivers and ADCs, and the number of ADCs that are combined determines the system dynamic range. For example, if the outputs of 100 ADCs were combined to form a beam, assuming that each ADC induces noise that is of equal amplitude and uncorrelated with the others, there would be a 20 dB increase in system dynamic range, compared to a single-receiver system using the same ADC.

## **6 MIMO Antennas**

Multi-Input Multi-Output (MIMO) technology has attracted attention in radar and wireless communications during the last decade. This is due to the dramatic improve in reliability, data throughput and link range as compared to single-antenna solutions in wireless communication without additional bandwidth or transmit power. [11]-[13] In radar MIMO systems possess significant potential for fading mitigation, resolution enhancement, and interference and jamming suppression. Fully exploiting these potentials can result in significantly improved target detection, parameter estimation, as well as target tracking and recognition performance. [14]

Unlike standard beamformers, which assume a high correlation between signals either transmitted or received by an array, the MIMO concept exploits the independence between signals at the array elements. In conventional radar, target scintillations are regarded as a nuisance parameter that degrades radar performance. The novelty of MIMO radar is that it takes the opposite view, it capitalizes on target scintillations to improve the radar's

performance. [12]

In contrast to standard phased-array radar, which transmits scaled versions of a single waveform, a MIMO system can transmit via its antennas multiple probing signals that may be chosen quite freely. This waveform diversity enables superior capabilities compared with a standard phased-array antenna.

MIMO radars offers the advantage of allowing the long dwell times required for target detection and clutter suppression, while maintaining continuous coverage of the entire volume. The second application area is aimed at detecting fast moving targets in clear air. Here MIMO radar offers the angular resolution performance of much larger phased array radars for a fraction of the weight and cost. MIMO also enables continuous coverage of the entire volume, facilitating detection of targets that may be missed by scanning beam systems.

The notion of MIMO radar is simply that there are multiple radiating and receiving sites, as shown in Fig. 14. [15] The collected information is then processed together. In some sense, MIMO radars are a generalization of multistatic radar concepts. The underlying concepts have most likely been discovered independently numerous times. [16]

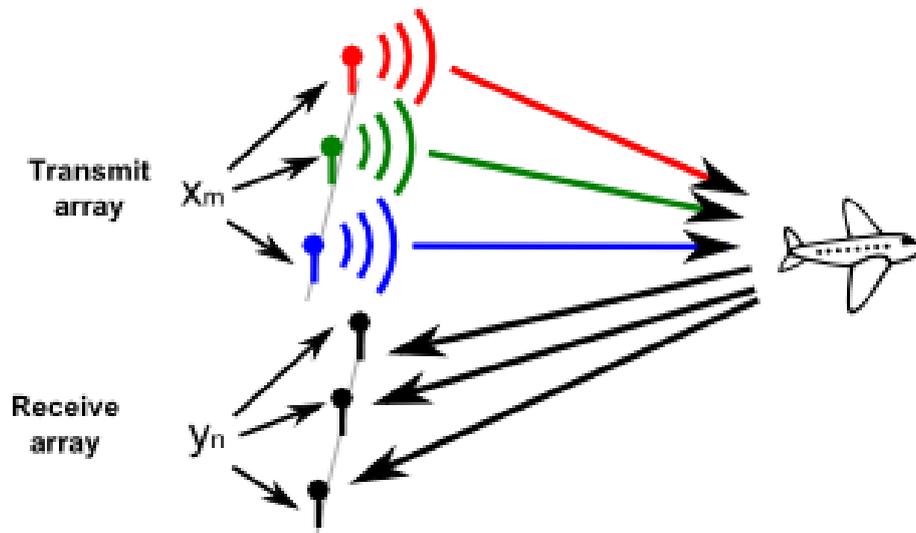


Figure 14: MIMO antenna array

There is a continuum of MIMO antenna concepts; however, there are two basic regimes of operation considered in the current literature. In the first regime, the transmit array elements (and receive array elements) are widely spaced, providing spatial diversity gain with independent scattering responses for each antenna pairing, sometimes referred to as *statistical MIMO*

*antenna*. In the second regime, the transmit array elements (and receive array elements) are closely spaced so that the target is in the far field of the transmit-receive array, sometimes referred to as *coherent MIMO antenna*. Here it is assumed that the target's scattering response is the same for each antenna pair, up to some small delay, which will be considered in more detail in this paper.

Beside the classification of MIMO systems concerning their antenna configuration, there exists the possibility to sub-divide them into three main categories by their modulation, precoding, spatial multiplexing, and diversity coding.

*Precoding* is multi-layer beamforming in a narrow sense or all spatial processing at the transmitter in a wide-sense. In (single-layer) beamforming, the same signal is emitted from each of the transmit antennas with appropriate phase (and sometimes gain) weighting such that the signal power is maximized at the receiver input. The benefits of beamforming are to increase the signal gain by constructive combining and to reduce the multipath fading effect. In the absence of scattering, beamforming results in a well defined directional pattern, but in urban areas beams are not a good analogy. When the receiver has multiple antennas, the transmit beamforming cannot simultaneously maximize the signal level at all of the receive antenna and precoding is used. Note that precoding requires knowledge of the channel state information (CSI) at the transmitter. Due to this circumstance precoding is mainly used in the wireless communication.

*Spatial multiplexing* requires MIMO antenna configuration. In spatial multiplexing, a high rate signal is split into multiple lower rate streams and each stream is transmitted from a different transmit antenna in the same frequency channel. If these signals arrive at the receiver antenna array with sufficiently different spatial signatures, the receiver can separate these streams, creating parallel channels for free. Spatial multiplexing is a very powerful technique for increasing channel capacity at higher Signal to Noise Ratio (SNR). The maximum number of spatial streams is limited by the lesser of the number of antennas at the transmitter or receiver. Spatial multiplexing can be used with or without knowledge of the transmit channel.

*Diversity coding* techniques are used when there is no channel knowledge at the transmitter. In diversity methods a single stream (unlike multiple streams in spatial multiplexing) is transmitted, but the signal is coded using techniques called space-time coding. The signal is emitted from each of the transmit antennas using certain principles of full or near orthogonal coding. Diversity exploits the independent fading in the multiple antenna links to enhance signal diversity. Because there is no channel knowledge, there is no beamforming or array gain from diversity coding.

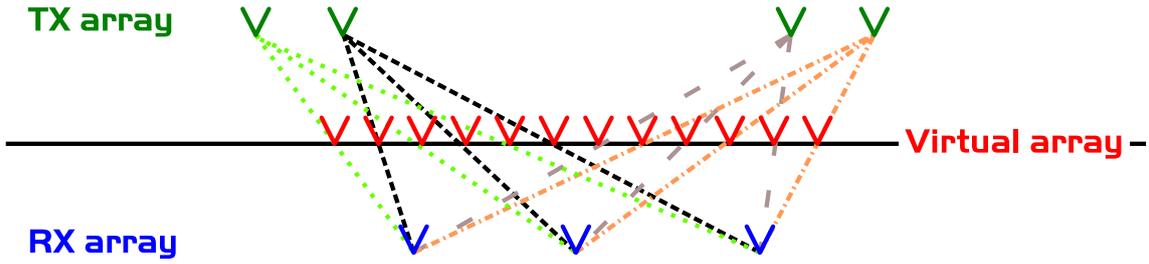


Figure 15: Example of a MIMO antenna, which forms a virtual linear antenna array

Spatial multiplexing can also be combined with precoding when the channel is known at the transmitter or combined with diversity coding when decoding reliability is in trade-off.

## 6.1 MIMO antenna - virtual linear antenna array

One of the main advantages of MIMO antenna is that the degrees of freedom can be greatly increased by the concept of virtual array. Consider an arbitrary transmitting array with  $N$  antenna elements and an arbitrary receiving array with  $M$  antenna elements. In general the  $n$ -th transmitting antenna is located at  $\mathbf{x}_{Tn}$  and the  $m$ -th receiving antenna is located at  $\mathbf{x}_{Rm}$ .

Fig. 15 shows an example with  $N = 4$  and  $M = 3$ . Imagine that the  $n$ -th transmitting antenna emits the waveform  $s_n(t)$  and that all emitted waveforms are orthogonal:  $\int s_n(t)s_m^*(t) dt = \delta_{nm}$ . In each receiving antenna, these orthogonal waveforms are extracted by  $N$  matched filters. Therefore, the total number of extracted signal is equal to  $M \cdot N$ . If there is a single-point-scattering target in the far field, exhibiting a reflection coefficient of  $\alpha(\xi)$ , the target response in the  $n$ -th matched-filter output of the  $m$ -th receiving antenna can be expressed as

$$y_{m,n}^{(t)} = \alpha(\xi) e^{j k_0 \mathbf{u}_t^T (\mathbf{x}_{Tn} + \mathbf{x}_{Rm})} \quad (31)$$

where  $\mathbf{u}_t \in R^3$  is a unit vector pointing toward the target from the radar station. One can see that the phase differences are created by both the transmitting and the receiving antenna locations. The target response is the same as the target response received by a receiving array with  $M \cdot N$  antenna elements located at

$$\{\mathbf{x}_{Tn} + \mathbf{x}_{Rm} \mid m = 0, 1, \dots, M - 1, n = 0, 1, \dots, N - 1\}.$$

We call this  $M \cdot N$ -element array a virtual array, as shown in Fig. 15. Thus, we can create an  $M \cdot N$ -element virtual array by using only  $M + N$  physical antenna elements. The re-

lation between the transmitting array, receiving array, and the virtual array can be further characterized by a convolution [17]. Define

$$g_T(\mathbf{x}) = \sum_{n=0}^{N-1} \delta(\mathbf{x} - \mathbf{x}_{Tn}) \quad (32)$$

and

$$g_R(\mathbf{x}) = \sum_{m=0}^{M-1} \delta(\mathbf{x} - \mathbf{x}_{Rm}) \quad (33)$$

These functions characterize the antenna locations in the transmitter and receiver. Because the virtual array has  $M \cdot N$  elements located at the virtual locations  $\{\mathbf{x}_{Tn} + \mathbf{x}_{Rm}\}$ , the corresponding function that characterizes the antenna location of the virtual array can be expressed as

$$g_V(\mathbf{x}) = \sum_{n=0}^{N-1} \sum_{m=0}^{M-1} \delta(\mathbf{x} - (\mathbf{x}_{Tn} + \mathbf{x}_{Rm})) \quad (34)$$

Comparing Eq. 32 - 34, one can see that

$$g_V(\mathbf{x}) = (g_T * g_R)(\mathbf{x}) \quad (35)$$

where  $*$  denotes convolution.

This technique is used in ARTINO [18] to build an virtual antenna array of 1408 elements by using only 44 transmit and 32 receive elements. The optimal location of the transmit and receive elements for this linear MIMO array is composed of  $M$  receiving elements which are nested between  $N/2$  transmit elements on both sides. The transmit elements are equally spaced of  $d$  and the gap between the RX and TX packages is  $d/2$ . Then the positions of the elements can be determined by the following equation:

$$\mathbf{x}_{nmk} = \frac{1}{2} \left[ \left( m - \frac{(M+1)}{2} \right) \frac{N}{2} d + \left( \frac{(M-1)}{2} \frac{N}{2} d + \frac{d}{2} + (n-1)d \right) k \right] \quad (36)$$

$m = 1 \dots M$ ,  $n = 1 \dots \frac{N}{2}$  and  $k = -1, +1$ .

From this the elements of the virtual antenna run from  $N_{virt} = -(MN-1)\frac{d}{4}, \dots, (MN-1)\frac{d}{4}$  in steps of  $\frac{d}{2}$ . This is the best of two possible solutions for an equispaced virtual array. The minimum number of real elements to synthesize a  $N_{virt}$ -element virtual array is  $N = M = \sqrt{N_{virt}}$ .

### 6.1.1 Geometrical consideration and signal model of a virtual linear antenna

Consider a linear MIMO antenna which is composed of  $N_{virt}$  elements, each centered at the  $y$ -axis and regularly spaced along this axis. The position of the  $n$ -th virtual antenna element on the  $y$ -axis is given by  $\frac{(N_{virt}-1)}{2} \cdot n - \frac{(N_{virt}-1)}{2}$  with  $n = 0, \dots, (N_{virt} - 1)$ . Fig. 16

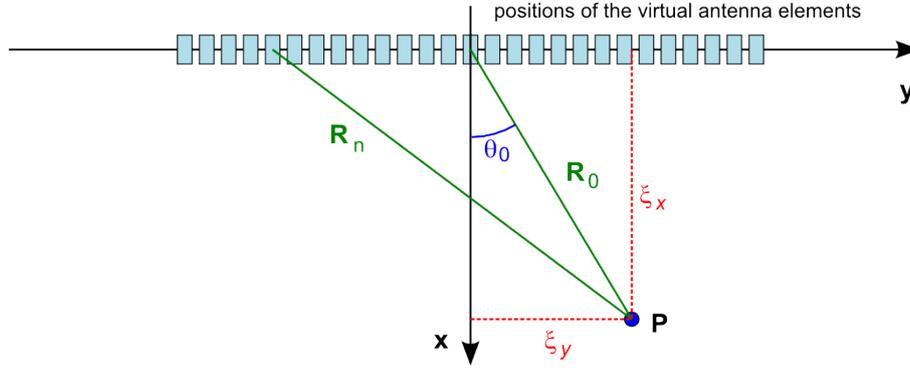


Figure 16: Geometry of a linear MIMO antenna

shows the geometry of the linear MIMO antenna. The distance  $d$  between the virtual antenna elements was determined by simulations in order to optimize the antenna beam (reduction of grating lobes) of the whole array.

To obtain a distinct assignment of each virtual antenna element, it will be necessary for the experimental system ARTINO that the real antenna elements transmit with a time multiplex from pulse to pulse. A point-scatterer  $P$  is positioned at  $\xi = (\xi_x, \xi_y, 0)'$  with the reflectivity  $\alpha(\xi)$  (see Fig. 16). The superscript  $'$  denotes the transpose operator. The signal assigned to the baseband signal is termed by  $s_{st}(t)$  with  $t$  the time. After reflection at  $P$ , the signal is received by the  $n$ -th virtual antenna element and can be written after quadratur-demodulation as

$$s(t, n, \xi) = \alpha(\xi) \cdot \mathcal{A}(\mathbf{u}) \cdot e^{-jk_0 R(\xi)} \cdot s_{st}(t - \tau(\xi)) \quad . \quad (37)$$

The two-way antenna characteristics (i.e. the multiplication of the illumination function of both the transmit and the receive element) in elevation-direction is denoted by  $\mathcal{A}$ . The look direction is equal for all antenna elements and given by  $\mathbf{u}$ . The two-way antenna characteristics in the  $(x,y)$ -plane will not be considered here, as the small antenna dimensions in this plane leads to a uniform illumination over the interesting area.  $k_0 = 2\pi/\lambda_0$  denotes the equivalent wave number and  $\lambda_0$  the wavelength of the carrier frequency. The term  $R(\xi)$  describes the range between the considered  $n$ -th virtual antenna element and the point scatterer  $P$ . The transmit signal is delayed by  $\tau(\xi) = 2R(\xi) / c$  with  $c$  as the speed of light. With the

substitution  $r = ct/2$ , the whole received signal is obtained by the sum of the radar echoes of all point scatterers within the antenna beam

$$s(r, n) = \iiint \alpha(\xi) \cdot \mathcal{A}(\mathbf{u}) \cdot e^{-j k_0 R(\xi)} \cdot s_{sr}(r - R(\xi)) \cdot d\xi \quad (38)$$

### 6.1.2 Beamforming operation

After pulse compression (focusing in range), the contributions of the scatterers located along a range line are still mixed with each other and appear in the same range cell. One particular principle of the linear MIMO antenna is the utilization of a digital beamforming operation in order to resolve the echoes of the scatterers in each range line. To clarify the following operations, range can be approximated as

$$R_{n0}(\xi) \approx R_0(\xi) + \frac{n^2 d^2}{2 R_0(\xi)} - n d \sin(\theta_s) \quad (39)$$

with  $\sin(\theta_s) = \xi_y / R_0(\xi)$ .  $R_0(\xi)$  defines the range to the center of the array and  $\theta_s$  denotes the incidence angle (see Fig. 16 for geometry). The quadratic term  $\frac{n^2 d^2}{2 R_0(\xi)}$  has to be compensated for focusing the data:

$$\begin{aligned} \bar{s}_r(r, n) &= s_r \cdot e^{j k_0 \frac{n^2 d^2}{2 R_0(\xi)}} \\ &\approx \iiint \alpha(\xi) \cdot p_r(r - R_{n0}(\xi)) \cdot e^{-j k_0 R_0(\xi)} \cdot e^{j k_0 \sin(\theta_s)} \cdot d\xi \end{aligned}$$

The range correction with  $R_{n0}(\xi)$  instead of  $R_0(\xi)$  in Eq. 40 causes a small de-focusing for large  $y$ .

Let's define the delay  $\phi(n, \theta) = k_0 n d \sin(\theta)$ . By varying the phase shift  $\phi$  between the different virtual antennas, the virtual antenna array can be steered to different directions. Focusing the antenna array to the direction  $\theta$  results in the following beamforming operation:

$$\begin{aligned} s_{r,\theta}(r, \theta) &= \sum_{n=-\frac{N_{virt}-1}{2}}^{\frac{N_{virt}-1}{2}} \bar{s}_r(r, n) \cdot e^{-j k_0 n d \sin(\theta)} \\ &\approx \iiint \alpha(\xi) \cdot p_r(r - R_{n0}(\xi)) \cdot e^{-j k_0 R_0(\xi)} \cdot \sum_{n=-\frac{(N_{virt}-1)}{2}}^{\frac{(N_{virt}-1)}{2}} e^{-j k_0 n d (\sin(\theta) - \sin(\theta_s))} \quad (40) \end{aligned}$$

After a few calculations and inserting the point spread function for the focussing angle  $\theta$

$$p_\theta(\sin(\theta')) = \frac{\sin(N_{virt} k_0 \frac{d}{2} \sin(\theta'))}{\sin(k_0 \frac{d}{2} \sin(\theta'))} \quad (41)$$

the following result is obtained:

$$s_{r,\theta}(r, \theta) = \iiint \alpha(\xi) e^{-j k_0 R_0(\xi)} \cdot p_r(r - R_{n0}(\xi)) \cdot \quad (42)$$

$$p_\theta(\sin(\theta) - \sin(\theta_s)) \cdot d\xi \quad (43)$$

Eq. 42 shows the back scattered signals compressed in range, azimuth, and angle with their point spread functions respectively. By switching all virtual antenna elements together, an antenna with a beamwidth of

$$\delta\theta = \frac{\lambda}{2 N_{virt} d} \quad (44)$$

can be formed. The sampling of the data with this narrow beam enables to resolve the radar echoes in  $y$ -direction.

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