Flight Test Verification of a Wake Vortices Model

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ABSTRACT

A simulation model for encounters of wake vortices has been developed at Saab in Linköping, Sweden. Distribution of incremental forces on the aircraft surfaces is calculated for different locations related to the vortices and also for different relative attitudes of the aircraft. Also, the response of the angle-of-attack and angle-of-sideslip vanes is calculated. For simulation, multidimensional tables are created describing the increments in forces, moments and vane angles as function of relative position and attitude.

The model, represented by table lookup algorithms for these particular tables as well as some scaling algorithms, was in the year 2000 added to an experimental version of a 6-DOF Gripen simulator. The results seemed promising because the response of the aircraft flying through the vortices was, seen from a pilot's view, realistic. However, it was evident that the model also needed to be verified with and, most likely, adjusted (scaled) to flight test data.

The need for accurate estimates of the aircraft's trajectory through the vortices requires special attention. Regardless of the ability to accomplish that accuracy in relative position between two aircraft, we still have to determine, also very accurately, where the vortices are, regarding cores and size. Local wind fluctuations and the self-induced motion of the vortices away from the flight path contribute with uncertainties in the range of a couple of meters. From that standpoint, we have too large uncertainties.

Our best solution so far is to use a "closed-loop" verification of the model where we use a subset of the output parameters to identify a two-dimensional position bias that gives us the most likely trajectory. We use the angle-of-attack and angle-of-sideslip vane signals for that purpose.

The result is not only a method for verification of a trailing vortex simulation model but also a tool to map out in detail where we have been in wake vortex penetration tests. With respect to the simplifying assumptions used in the design of the current model, the agreement with flight test data is surprisingly good.

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1.0 NOMENCLATURE

B	Label for "Wake encountering aircraft"
b_0	Distance between vortex centers
C_C	Side force coefficient
C_L	Lift coefficient
C_l	Roll moment coefficient
C_m	Pitch moment coefficient
C_N	Normal force coefficient
C_n	Yaw moment coefficient
C_T	Tangential force coefficient
e	Error, residual
FCS	Flight Control System
G	Label for "Vortex generating aircraft"
GPS	Global Positioning System
INS	Inertial Navigation System
M	Mach number
r_c	Core radius
t	Time
U	Model input
U_m	Measured model input
w_i	Circumferential velocity
x, y, z	Position coordinates
Y	Model output
Ym	Measured data corresponding to model output
Δt	Time separation between aircraft
Γ	Vortex circulation
Γ_0	Root circulation
α	Angle of attack
α_l	Streamline incidence at left angle-of-attack vane position
α_r	Streamline incidence at right angle-of-attack vane position
β	Angle of sideslip (Not applicable in eq. 1)
β_i	Streamline incidence at angle-of-sideslip vane position.
χ^A, γ^A, μ^A	Flight path angles with respect to the ambient air mass.
ϕ	Roll angle
θ	Pitch angle
ψ	Yaw angle

2.0 INTRODUCTION

Wake vortices have a direct physical relationship with aerodynamic lift. Thus they can be found behind every airplane that flies. The strength of the vortices is proportional to the lift and inversely proportional to the airspeed. Generally described by the term "wake turbulence", these vortices can be very hazardous to aircraft passing areas where other aircraft has flown recently, up to several minutes earlier. As the vortices have a self-induced motion through the air and also travel with the wind, it can be rather hard to predict where the vortices are located. The general rule concerning wake turbulence is "avoid it", especially if the "other" aircraft is heavier. As the intensity of the vortices is higher when the generating aircraft is flying slow than when it is flying at cruising speed, it is also in operations around airports pilots have to be most aware of the risk. Most of the research concerning wake turbulence has also been focused on the long-term life-cycle of the vortices, methods to predict where they go and methods to shorten their life-span. This paper deals with a different approach to the subject. Here we discuss rather young vortices, before the so called Crow instability has a significant influence, and we discuss fighter aircraft that hit them. In dogfight scenarios, we have to recognize the fact that wake encounters will happen, often at rather short intermediate distances between the aircraft. Fighter development over the last decades has probably led to higher probability for wake encounters due to:

- Smokeless engines – no visual cues in the wake.
- Increased maneuvrability and tighter turns.
- Carefree maneuvring capability – it is easy for the pilots to achieve optimum performance.

In Sweden, both the Viggen and Gripen aircraft underwent wake encounters in their flight test programs. While the tests in the Viggen gave some interesting results like an engine flame-out and a disintegration of the uppermost part of the fin, the tests in the Gripen did not expose any weak spots. Experience from air force service also confirms that a vast majority of wake encounters pass with no signs of problems. However, a few of them have led to violent angle-of-attack overshoots and the loss of a Gripen in 1999 and a spin incident in 2003 were both triggered by wake encounters. Thus, we had a behavior of the aircraft in a small portion of the wake encounters that we did not discover during earlier flight tests. A fix to the problem is underway. However, the lesson to be learned, applicable for Gripen and probably for other modern fighters, is that when it comes to verification of the aircrafts ability to withstand wake encounters, a limited number of flight tests is not enough. In the same way as we use simulations and flight tests together for determination of safe maneuvring limits, we also have to use simulations for a more thorough scanning of wake encounters. Starting with a model that to some extent describes the behavior physically, it is possible to draw more far-reaching conclusions from a limited number of flight tests. We can describe the test results in terms of model parameters. In the world of simulations, it is then possible to repeat different setups with small variations of the inputs and make thousands of virtual wake passages.

This paper includes a brief presentation of a model that we have developed for Gripen, followed by an overview of how the "vortex aerodata" model interacts with other part models in the simulation of Gripen's flight dynamics. One chapter contains a description of how the model is used in flight test data analysis, and particularly we discuss the problem concerning the estimation of the relative position of the aircraft in the vortex field with sufficient precision. Some results are also presented.

3.0 MODEL DESCRIPTION

The history of the development of the "vortex aerodata" model for Gripen is in brief:

Year 2000 - 2001: Demonstration of the possibility to simulate wake encounters in a Gripen simulator.

Year 2003 – 2004: Investigation of the model's feasibility for further development of the flight control system, aiming to improve Gripen's durability in wake encounters. This phase also included development of flight test analysis methods.

Year 2004 – 2005: Development of a wake vortices transient reduction function in Gripen. First "production" version of the "vortex aerodata" model has been defined during this phase.

Figure 1. SAAB Gripen. Positions for left angle-of-attack (α) vane, angle-of-sideslip (β) vane and center of gravity (CG) are indicated.

The "vortex aerodata" model was designed to work as an addition to the existing aerodata model for Gripen. Thus the vortex aerodata model should only describe the additional force and moment vectors due to the vortex flow, while the ordinary aerodata model described the rest of the aerodynamics. The increment in the sensor input for angle of attack and angle of sideslip due to the vortices was also included in the model. Gripen has three vanes for measurements of local incidence angles. They consist of the left and the right angle-of-attack vane and the single angle-of-sideslip vane (fig. 1). No other effects are described by the current model. The basic philosophy in the development was to use simple assumptions, a limited set of input variables and a limited set of output variables. The following section is a brief description of how the model is built. See [1] for a more detailed description of the model design.

In the description of the model and the flight test data analysis, the letter "B" denotes the aircraft encountering the vortices while the letter "G" is used for the aircraft generating these vortices.

When an aircraft moves through the air and generates lift, there is a downwash behind the aircraft. At a distance corresponding to a few spans behind the aircraft, the downwash has almost entirely rolled up into two counter-rotating vortices (fig 2). In this context, we prefer the term "wake vortices" in favor of "wake turbulence". Turbulence is a random process. The dominating effects in the wake are attributed to the influence of the ordered flow in the two vortices, not to any disordered flow, even if the latter can be felt as vibrations.

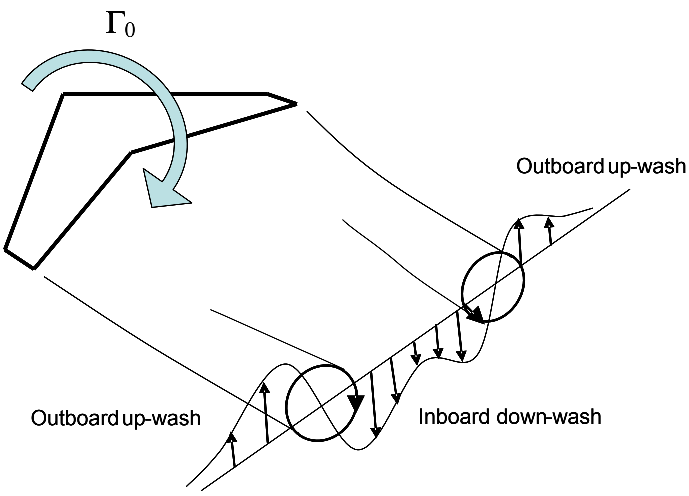

Figure 2. Roll-up of wake flow

Γ_0 is the root circulation for aircraft G. That circulation propagates to the vortices. The vortex circulation Γ is assumed to start from Γ_0 and then exponentially decay with longitudinal coordinate x behind aircraft G.

A typical velocity profile for the flow in a single vortex is shown in fig 3. In the radial direction, the flow can be divided into a viscous part near the center where the air behaves almost like a rigid cylinder and an outer part where the velocity profile is close to that of an ideal inviscid potential vortex. The velocity profile used for creation of the simulation model is according to the formula below, named "expanded Lamb" profile because it is a modification of a classic vortex theory described in works by Sir Horace Lamb. w_i is the circumferential velocity at the distance r from the vortex. r_c is the core radius. The expanded Lamb profile is adapted to the results from the experiment with a B-747 reported in [2].

$$w_i = \frac{\Gamma}{4\pi \cdot r} \cdot \frac{\theta'}{1+\theta'} \cdot \left(\frac{x}{\left(x^2 + \beta^2 r^2\right)^{1/2}} + 1 \right) \qquad (1)$$

$$\theta' = \left(\frac{r}{r'_c} \right)^2, \quad r'_c = \theta_c^{-1/2} \cdot r_c$$

$$\beta^2 = 1 - (M_G)^2$$

$$\theta_c = 1.2565, \quad r^2 = y^2 + z^2$$

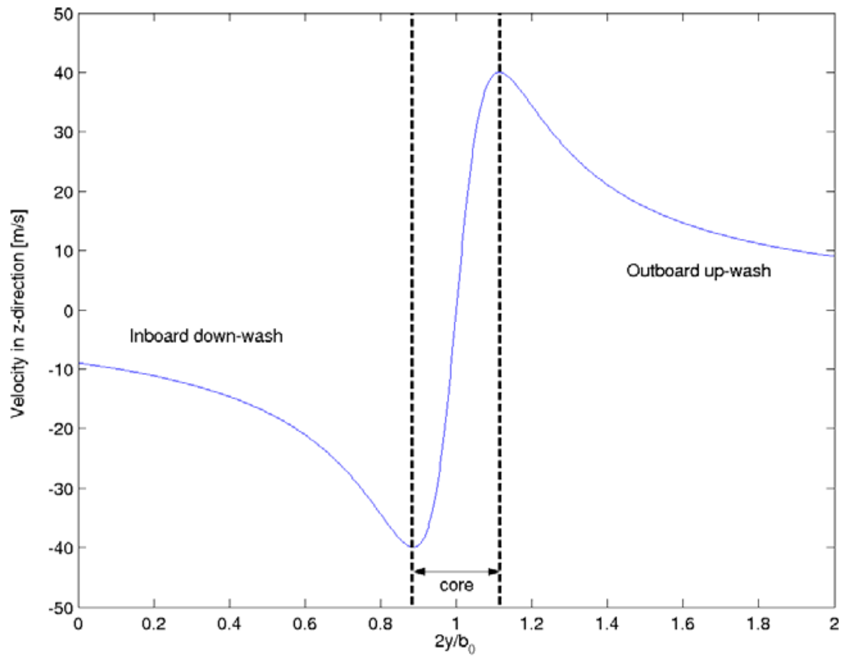

Figure 3. Velocity profile for one vortex calculated at a distance of 100 m behind a Viggen aircraft flying at C_L = 1.0, Mach number = 0.5 and altitude = 4000 m. b_0 = 8 m.

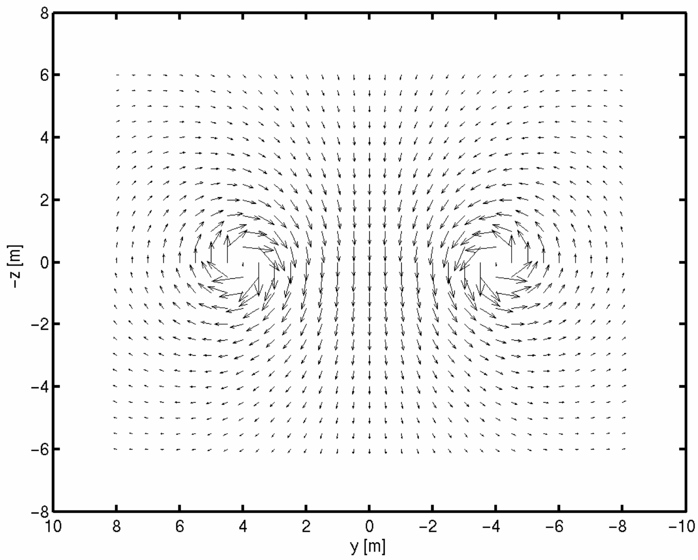

Figure 4. Velocity vector plot, two vortices.

Fig 4 shows a vector plot of the flow around a vortex pair. An important parameter is the distance between the vortex centers. That distance is usually somewhat less than the span of the vortex generating aircraft and it is assumed to be fairly constant for young vortices.

For calculation of forces and moments, linear potential theory has been applied on a panel model with approximately 500 panels (fig. 5). Intentionally, the panels together constitute a “plank” model with no interior volume. All surfaces are aligned with the airspeed vector related to the ambient “vortex-free” air. In other words, free-stream angle-of-attack and angle-of-sideslip has both been assumed to be zero. When used together with boundary conditions derived from the vortex flow model, the force on each panel is therefore attributed to the vortex flow only.

For calculation of the increment in vane angles, a 2D cross-section model with 2D doublet panels is used. (fig. 6). Each panel has counter-rotating elementary vortices at the ends. Cross-flow boundary conditions are created at all panel midpoints by the imposed wake flow. The local crossflow at panel joints is then derived by smearing the resulting corner circulation over the corresponding arc lengths. The method was validated using wind tunnel measured streamline inclinations in uniform flow.

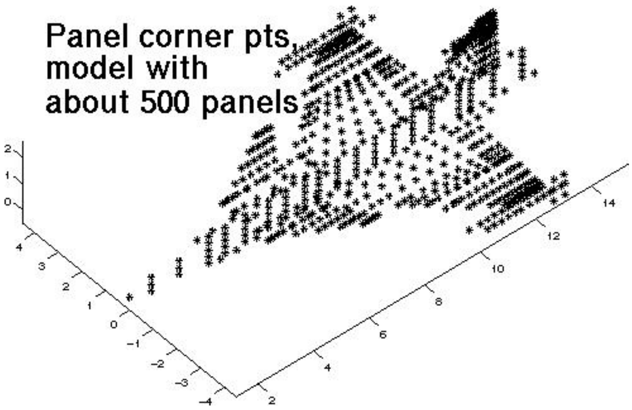

Figure 5. Panel model of Gripen.

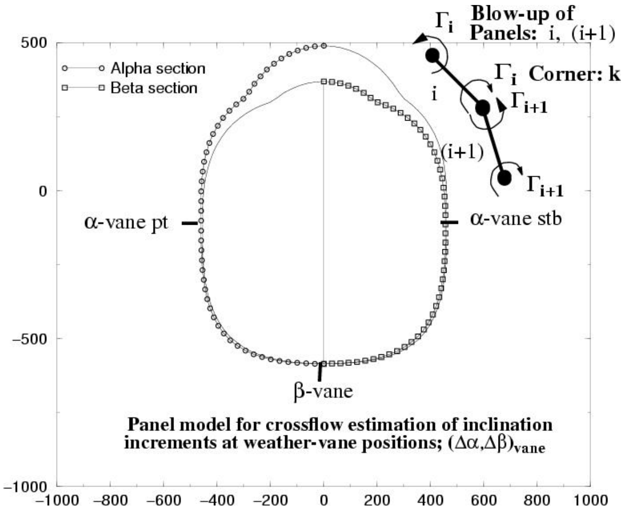

Figure 6. Fuselage cross-sections at α/β-vane locations

The program structure is shown in fig. 7. CAMBER calculates the boundary conditions, WB calculates forces and moments and SECT calculates streamline inclinations at vane positions.

For the creation of the look-up tables which together constitute the vortex aerodata model used in simulations, each point in the tables identified by a combination of the input variables x,y,z,ϕ,θ and ψ is an "aero case" to be computed with the programs mentioned above.

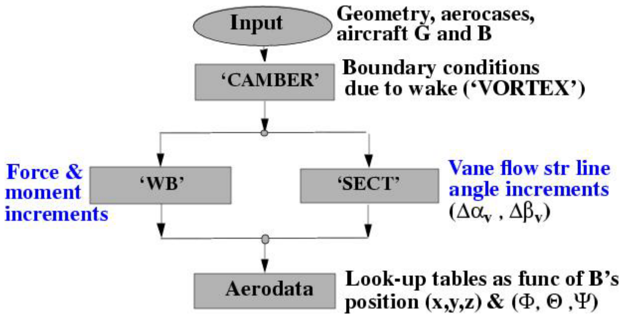

Figure 7. Program system for creation of look-up tables.

A complete set of inputs for an "aero case" also include reference speeds for both aircraft, reference atmosphere conditions, a lift coefficient combined with a reference wing area for the vortex generating aircraft and the initial distance between the vortex centers. These variables are kept constant in the creation of the table lookup model. A table lookup model is therefore created for a certain combination of aircraft types. "A Gripen behind a Gripen" has been the usual scenario in recent simulations. For analysis of some flight tests, "a Gripen behind a Viggen" model was also built.

The tabulated functions also yield a reference condition in terms of airspeed of aircraft B, the airspeed of aircraft G, the lift for the vortex generating aircraft and the air density. The output is then supposed to be scaled to actual conditions.

Fig. 8 shows the output variables as functions of the y coordinate while the other input variables are kept constant. The results are representative for a Gripen behind a Viggen, altitude 2000 m, Mach number is 0.5 for both aircraft and the lift coefficient for the vortex generating aircraft is 1.0. The distance between the aircraft, measured along the flight path of the vortex generating aircraft, is 100 m. $z = 0$, thus the aerodynamic reference point of aircraft B is at the same z coordinate as the vortex centers. The relative attitude angles are all zero.

The diagrams in the top of the figure show force increments in coefficient form. The diagrams in the middle of the figure show moment increments in coefficient form. The diagrams in the bottom of the figure show the increments in vane angles. When the distance to the vortices grows, all effects are diminishing asymptotically to zero. The outer boundary for the tabular models for a Gripen behind a Gripen as well as for a Gripen behind a Viggen is ±30 m in y and z directions.

Another limitation chosen for the tabular models is that relative pitch angle is between -45° and 45° and the relative yaw angle is between -30° and 30°. If the wake encounter shall result in large attitude or angle-of-attack deflections, the path of the incoming aircraft must be close to parallel to the vortex trails. The pitch and yaw angle boundaries then also allow some margin for the incoming aircrafts angle-of-attack and for the transients caused by the vortices.

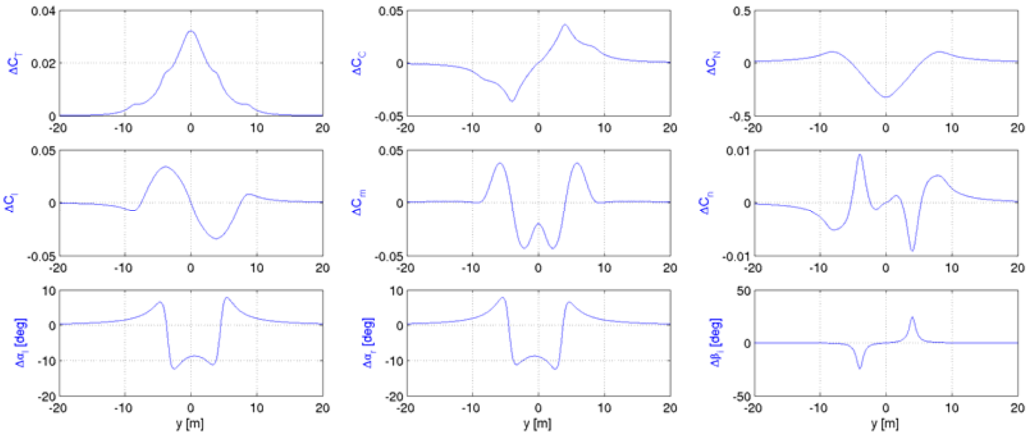

Figure 8. Induced force coefficients, moment coefficients and streamline inclinations at vane positions, traversing aircraft "B" through vortex cores behind aircraft "G".

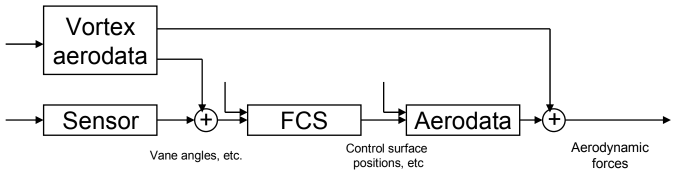

Figure 9. Generic block diagram for a part of the Gripen flight dynamics simulator

4.0 MODEL RELATIONSHIP

The vortex aerodata model was designed as an add-on to an existing application for simulation. Fig. 9 shows the dependencies between some selected blocks in the Gripen simulator. The "sensor" block calculates the sensor input to the flight control system. The sensor inputs include left and right angle-of-attack vanes and the angle-of-sideslip vane. The signals from these vanes correspond to the local flow incidence at the vane positions and the "sensor" block calculates the local incidence angles from the free-stream airspeed vector and some other variables. When the vortex aerodata model is included, the free-stream airspeed vector is still defined with reference to the ambient air "outside" the vortex field. The "vortex aerodata" block delivers the additional vane signal increments due to the vortex flow. The addition is actually done in terms of local airspeed cross-flow components rather than in terms of angles.

The vane angles are input to the "Flight Control System" (FCS) block. There are also other input signals to this block, e.g. the pilot's stick commands. Output from the "FCS" block includes the control surface positions.

The ordinary "aerodata" block contains a detailed description of the aerodynamic force's and moment's dependencies of a number of variables, among them the control surface positions. This block is based on the assumption that the ambient flow field is homogenous. Thus the ambient free-stream airspeed vector is part of the input to this block. The "vortex aerodata" block delivers the increments to the force and moment due to the wake vortices.

The wake vortices aerodata model has been demonstrated in a manned simulator (see fig 10 where the vortices are visualized with red-white banners). The model will be included in Gripen training simulators, both with and without visual cues (smoke).

Figure 10. Manned simulation of wake encounter

5.0 FLIGHT TEST ANALYSIS

The basic concept for model verification, in some contexts also described as model validation, is shown in fig. 11. We use a measured input U_m and compare the output Y with the corresponding measured signals Y_m. The difference, the residual e, is a measure how well the model can predict the output. The wake vortices aerodata model is a quasi-static model. There is no dependency of what happened in the past and no feedback loops inside the model. Thus, if it was possible to make a direct comparison according to the basic concept, the output could be compared on a sample-by-sample basis. However, in order to facilitate physical interpretations of the results it is useful to display data as time histories of the test runs.

The input to the table lookup function of the wake vortices aerodata model consist of $U = (x,y,z,\phi,\theta,\psi)$. The output from this function yield a reference condition and it has to be scaled with a factor that depends on parameters governing the strength of the vortices, i.e. the circulation Γ, and also on the airspeed of aircraft B and the air density. The circulation Γ and the path and orientation of the vortices has to be estimated from data for aircraft G. Later on in the analysis we will have to make adjustments in the position variables y and z. Nevertheless it is important to make a good first estimate of the input to the table lookup function. This step includes transformation between various reference postitions in the aircraft like the aerodynamic reference point in aircraft B, the origin for he vortex trails in aircraft G and the GPS antenna position.

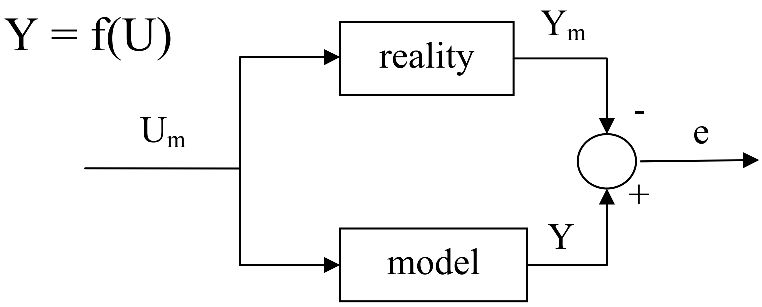

Figure 11. Generic block diagram for model verification

A few basic physical facts about the behavior of the vortices need to be considered. The first one is that when the vortices have been produced by a passing aircraft, they will be travelling under influence of the ambient wind. Therefore we transformate positions and velocities for both aircraft to a common reference system which for each test run travels with a representative wind speed and direction. In our analysis, we estimated wind speed as the difference between the airspeed vector and the ground speed vector in measured data from aircraft B before and after the wake vortex transients and picked a representative mean value for each run. Vertical wind components are neglected. When we use the term "flight path" in the description below, we implicitly refer to wind corrected flight paths.

The vortex pair also has a self-induced motion relative the ambient air. As long as we deal with comparatively young vortices with age up to a few seconds, the movement is principally in the z direction in a coordinate system fixed to the flight path of the generating aircraft. The velocity of this movement is proportional to the circulation Γ in the vortices. The position of the vortices in the wake is determined by solving a set of nonlinear differential equations containing the circulation, the vortex positions and the vortex velocities. A diagram over the vortices' self-induced travel in free air is shown in fig. 12.

This estimation of the position of the vortices relative the flight path of aircraft G is dependent of parameters and assumptions in the model. Thus it can be regarded as an extension of the model for flight test data analysis.

Before we establish a coordinate system tied to the flight path and estimate the position of the vortices, we have to consider the fact that the flight conditions of aircraft G are probably not constant. A coordinate system is positioned on the flight path of aircraft G in a way so the x coordinate for aircraft B is zero in that system. To make this definition almost unambiguously, we have to add that aircraft B shall be so close to the origin of this system as possible, and in order to make it feasible for automated data processing we only do it for limited time segments covering each passage. To be noted here is also the focus we have on "near-parallell" wake vortex passages. This coordinate system is oriented according to the three rotational components for the flight path, χ^A, γ^A and μ^A. The superindex "A" denotes flight path angles relative to a system that travels with the average wind, not flight path angles relative to the Earth. It is travelling along the flight path with a time separation Δt from aircraft G and this time separation is itself a function of time. Test data from aircraft G that originally are time-synchronized with test data from aircraft B are now time-shifted so the data governing the properties of the vortices at the time t, when we observe the behavior of aircraft B, should yield what happened in aircraft G at time t - Δt (fig. 13).

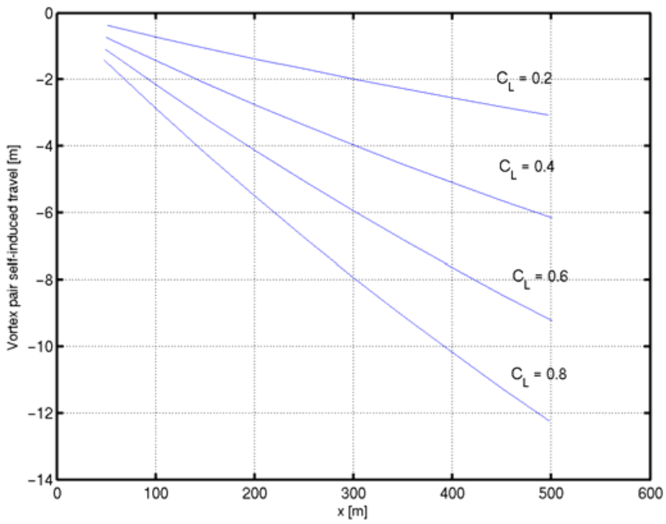

Figure 12. Theoretical estimations of the self-induced travel of the vortex pair behind a Viggen flying at M = 0.5.

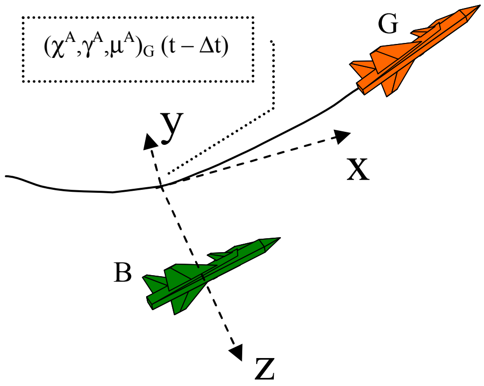

Figure 13. Coordinate system for analysis of wake encounter.

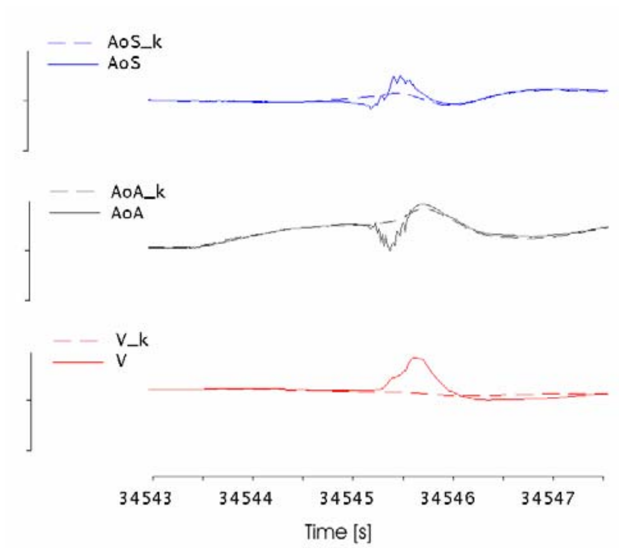

Figure 14. Time history plot from a wake encounter. Plotted variables are from top to bottom angle of sideslip, angle of attack and true airspeed. Solid lines represent measured data. Dashed lines represent output from kinematic intergation.

An important step in the analysis is the kinematic integration of the airspeed vector, i.e. true airspeed, angle of attack and angle of sideslip. It is accomplished by integration of INS data and with initial conditions and biases applied so the integrated airspeed vector matches the measured airspeed vector when the influence from the vortices is negligible. This is done for each test run. The output from this step represents the airspeed related to the ambient homogenous flow field and it does not include the influence the vortices have on the airdata sensors. Fig. 14 contains a comparison between measured data and the integrated data from a sample test run. In the logged data, there is a footprint from the wake vortices. There are some transients also in the integrated data. The free-stream airspeed vector relative the aircraft changes due to the motion of the aircraft induced by the vortices.

The purpose of the kinematic integration can be explained via a closer look on the simulation model structure (fig. 9). In the simulation tool, the input to the ordinary aerodata model is assumed to include the flight condition related to the ambient undisturbed air. The same assumption applies for the input to the ordinary sensor model. As there is no possible way to make direct measurements of the force and moment increments or the vane signal increments due to the airflow in the vortices, we have to use the ordinary aerodata model and the ordinary sensor model to derive these increments from estimated total aerodynamic forces and moments and from the logged vane signals.

The experimental results presented in this paper are derived from flight test data recorded during wake encounters flown in May 2001. The test aircraft exposed to vortices ("aircraft B") was one of the Gripen prototypes, namely 39-3. The lead aircraft ("aircraft G") was an instrumented Viggen (37.347). The primary purpose of these tests was an investigation of the mechanical environment in the aircraft. At that time, the work on the "vortex aerodata" model was halted, but data were logged on a "piggy-back" basis for later evaluation.

Most simulations with the model represent "Gripen behind Gripen". However, for analysis of the tests, a "Gripen behind Viggen" table look-up model was created. Later, when it became interesting to vary model parameters in the analysis, a "generic" model consisting of the codes CAMBER, WB and SECT was also used in the analysis. Thus we did the aerodynamic calculations with the panel models in every sampled point that was close enough to the vortices. This approach is acceptable for post-flight analysis when run times are not critical.

Considering the sensitivity of the model to relative position between the aircraft and the vortices, it should be no surprise that even with accurate GPS measurements and the corrections for the estimated wind- and self-induced travel of the vortices, measurements will not be accurate enough. A conservative estimation is that we in the analyzed data have uncertainties in the range of 5 – 10 m in the relative position and that is far too much for a direct comparison between model output and measured data. The work-around chosen here is to close a loop around the model and adjust the y and z coordinates with a bias identified for each test run so the vane angle "footprints" are as close as possible to model estimations. With this approach, it is possible to explain the transients in the wake encounters to a great extent by means of the model. Naturally, more statistics are needed with this approach to verify parameters like the circulation Γ in the vortices. Fig. 17 shows the block diagram structure for the revised approach. The "weights" block symbolize that the optimization algorithms operate on a subset of the residual representing the error in the streamline inclinations at vane positions.

Fig. 16 shows the computer tools developed for interactive analysis of the wake encounters. The tools are based on a combination of MATLAB[1] and in-house developed programs. Adjustments of the y and z bias can be made both manually and via optimization algorithms.

[1] MATLAB is a trademark of The Mathworks, Inc.

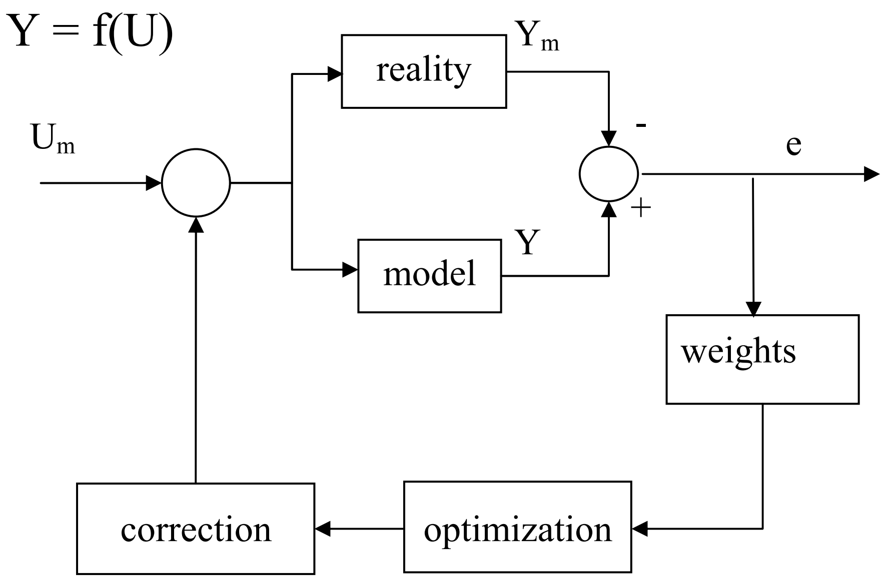

Figure 15. Block diagram for model verification including an input error optimization loop.

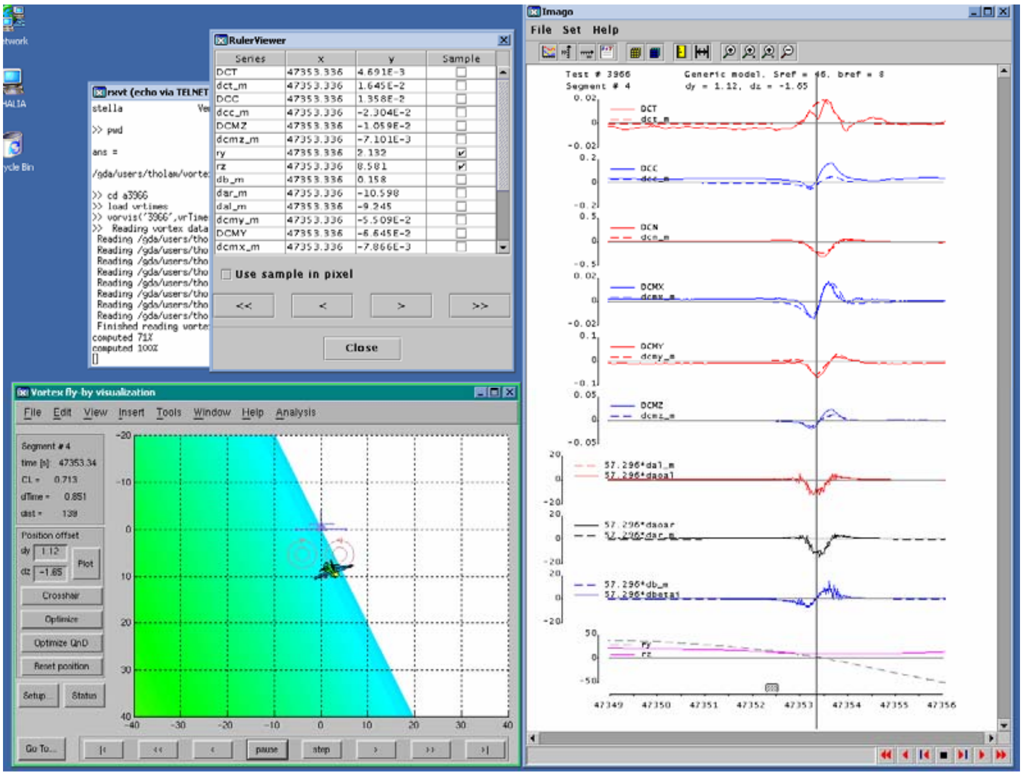

Figure 16. Interactive computer program for analysis of wake encounters

6.0 RESULTS

Fig. 17 shows a time history plot for a sample test run expressed in the model output parameters. The increments in forces and moments are plotted in coefficient form. The vortices are assumed to have a distance of 8 m which is approximately 80 % of the Viggen's wing span.

Considering the simplicity of the model, the conformity to measured data in this run is surprisingly good. All together, 21 test runs has so far been analyzed and in most cases the agreement is pretty good. Wake encounters at Mach numbers from 0.35 to 0.8 have been analyzed. Fig. 17 indicates that some deviations occur in the side force coefficient C_C and this is typical for some test runs. However, in the comparisons, we also have to consider that there are also some errors in the output from the ordinary aerodata model.

There are also a few test runs where rolling and yawing moments from the model deviate significantly from measured data. It seems however likely that some of these errors can be explained being due to a shift in the vortices positions due to backward interference from the incoming aircraft. Though this effect has been observed in earlier flight tests with Saab aircraft, it was excluded from the assumptions for the sake of simplicity.

7.0 CONCLUSIONS

- The complexity and agility of modern fighters makes it necessary to have a detailed understanding of the dynamics in passages of wake vortices.
- The “vortex aerodata” model is a valuable addition to simulation facilities for design and analysis of the flight control system for modern fighters.
- With respect to the simplifying assumptions used in the design of the current model, the agreement with flight test data is surprisingly good.
- For flight tests with Gripen, a comparison of model data and measured vane angles is a way to get accurate estimates of the trajectory through the vortices.

8.0 ACKNOWLEDGEMENTS

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9.0 REFERENCES

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[2] Burnham D.C. et al. 1978 Ground-Based Measurements of a B-747 Aircraft in Various Configurations. *US. Department of Transportation Report FAA-RD-78-146.*

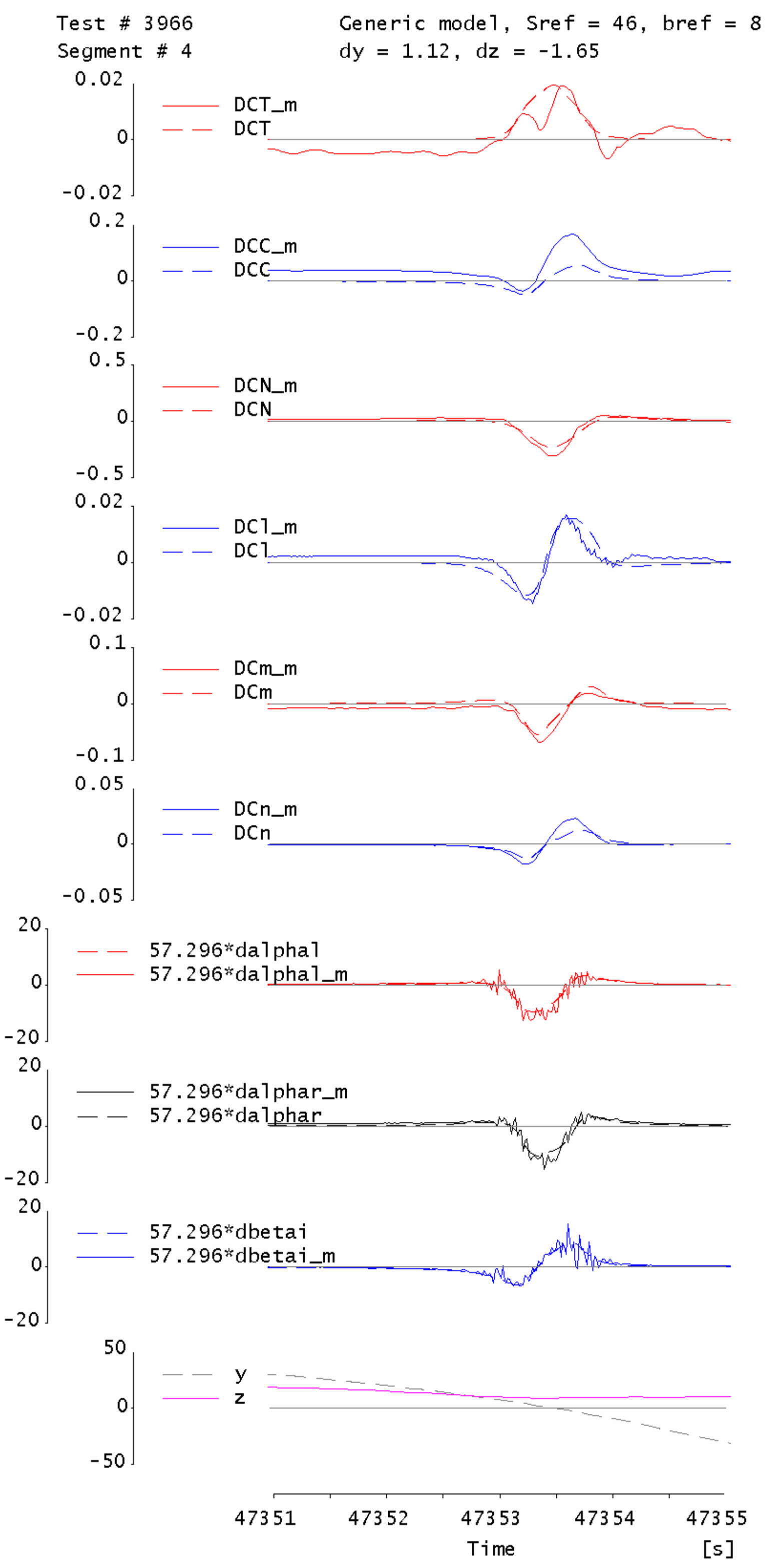

Figure 17. Comparison between model output (no suffix) and data based on measurements (suffix "_m") for a sample test run. Inputs y and z are plotted at the bottom of the diagram.